Structuring the system of didactical suitability criteria for mathematics instruction processes

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Research in mathematics education aims to produce knowledge about the various aspects involved in mathematics teaching and learning processes. The study of research handbooks, journal articles, monographs and conference proceedings on any mathematical topic suggests the vast amount of knowledge available and the difficulty involved in its organization and understanding. From these investigations, the need for criteria that allow establishing research relevance and adequacy regarding the design, implementation and evaluation of mathematics instruction is clear. In this paper, we propose a system of categories to classify the didactical suitability criteria by applying some theoretical notions from the Onto-semiotic Approach (OSA) of mathematical knowledge and instruction (Godino, Batanero y Font, 2007; 2019).

PROBLEM AND BACKGROUND:

The notion of didactical suitability has been introduced in previous works (Godino, 2013) as a tool for the design and evaluation of mathematical instruction processes. OSA principles are assumed for its development, in particular, it is considered that the didactics of mathematics has a technological component and, therefore, it should address the problem of optimizing the development of teaching and learning processes. Because of didactic research it is accepted that knowledge is obtained which is translated into preferential criteria on what mathematics should be taught and how, according to the contexts, circumstances and people involved. The problem of how to formulate and categorize suitability criteria is raised in order to have an organized system that takes into account the different dimensions and components that characterize the instructional processes. This system can be specified in guides that support systematic reflection on the practice of teaching, and therefore, they are resources for teacher education.

The confrontation of the system of categories and criteria proposed in Godino (2013) with other models suggests its revision and possible updating. This is what Breda, Font, and Pino-Fan (2018) did by introducing important changes in the epistemic dimension, taking into account the MQI model (Hill et al., 2008). However, we believe that it is necessary to deepen the topic and develop a system of categories and criteria that take advantage of the possibilities offered by the OSA tools. In particular, for the epistemic and cognitive dimensions it is necessary to put in the foreground the notion of partial meaning (sense) of a mathematical object and the articulation of the different partial meanings in a global meaning.

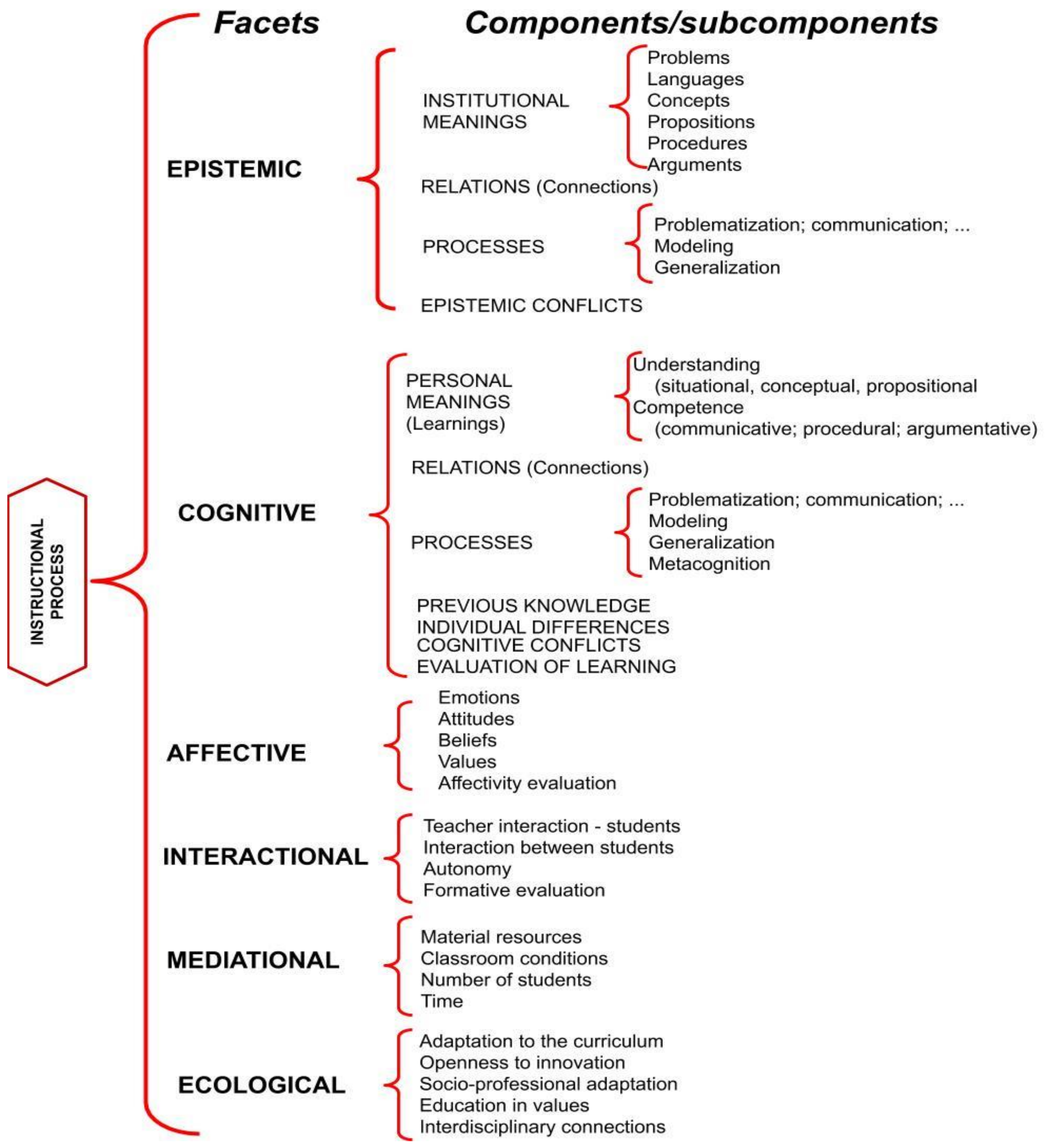

Fig. 2. Categories of didactical suitability criteria (CDSC model)

MQI model		CDSC/OSA model
Standard	**Codes**	**Dimension/component**
1. Richness and development of the mathematics	1.1. Presence of multiple models in the classroom (symbolic and visual representations,...), coordinated and linked together	Epistemic / situations-rules
	1.2. Links between the multiple models are established	Epistemic / relationships
	1.3. Mathematical explanations	Epistemic / arguments
	1.4. Mathematical justifications: they are endowed with sense and meaning	Epistemic / situations, relationships
	1.5. Speaking explicitly about the mathematical language, reasoning, and practices	Epistemic / languages, arguments
	1.6. Patterns and generalizations	Epistemic / mega process
2. Responsiveness to student ideas	2.1. The productions of the students are interpreted	Interaction / teacher-student interaction
	2.2. Student errors are corrected and exploited	Interaction / teacher-student interaction
3. Connecting classroom practice to mathematics	3.1. The students' work is connected to mathematical ideas or procedures	Mediational / resources Cognitive / relationships Affective / interests
	3.2. The mathematics of the lesson is developed during the class segment or outside	Mediational / time
	3.3. Instructional time is invested in mathematics	Mediational / time
4. Language	4.1. Conventional notation	Epistemic / languages (symbolic)
	4.2. Technical language	Epistemic / languages Epistemic / rules
	4.3. The notion or terms are used simply during instruction or explicitly talked about its meaning	Epistemic / rules Epistemic / language Interaction / time
	4.4. Use of ordinary language to express mathematical ideas	Epistemic / language Epistemic / relationships (with concepts, ...)
5.Equity	5.1. Access of all students to school mathematics	Cognitive / adaptations
	5.2. Sensibilidad a las diferencias individuales en conocimientos previos	Cognitive / individual differences, previous knowledge
	5.3. Sensitivity to individual differences in prior knowledge	Affective
	5.4. Opportunity for students to participate and learn	Affective
6. Presence of unmitigated mathematical errors	6.1. Errors in the mathematical content presented	Epistemic / cognitive: rules, arguments, relationships Epistemic conflicts
	6.2. Imprecision in the language or notation	Epistemic / cognitive: languages
	6.3. Lack of clarity in the presentation of mathematical content	Epistemic / cognitive: rules, arguments, relationships
7. Overall student participation in meaning-making and reasoning. (Students' cognitive level)	7.1. The students provide explanations	Cognitive / learning (argumentation)
	7.2. Students elaborate mathematical reasoning and questions	Cognitive / learning (situational, argumentative)
	7.3. Students work with contextualized problems	Cognitive / learning (modelling)
	7.4. Students activate the expected cognitive demand	Cognitive / learning (personal meanings)

Fig. 3. Comparison of MQI and CDSC models

METHOD:

The revision of the system of categories for the epistemic and cognitive dimensions is done taking into account the OSA onto-semiotic configuration tool (Fig. 1), as well as the notion of global meaning of a mathematical object. The comparison of the CDSC and the MQI models is done by projecting the respective category systems.

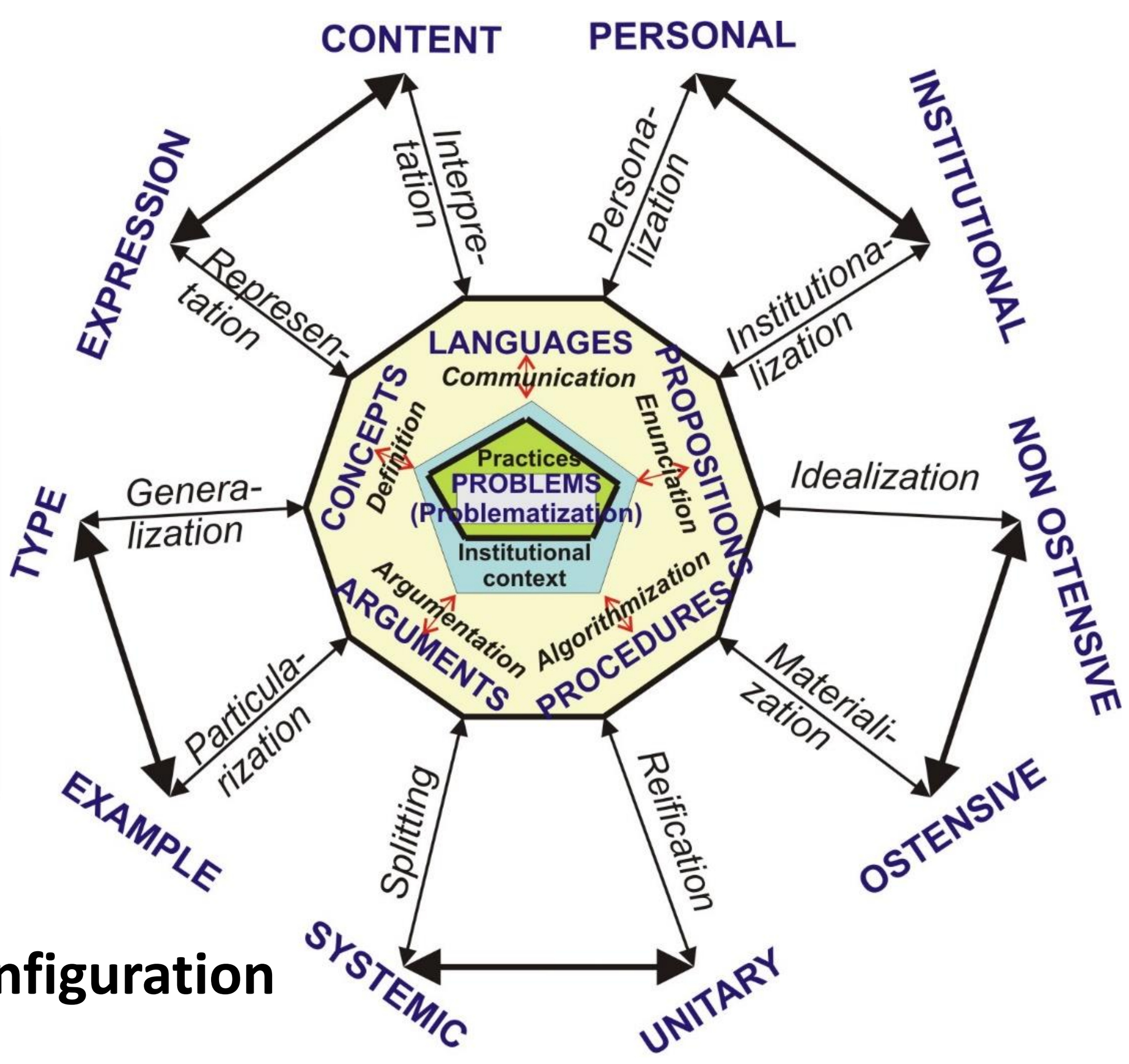

Fig. 1. Onto-semiotic configuration

RESULTS:

In Fig. 2, we summarize the system of hierarchical categories that we propose to classify suitability criteria and guide their formulation. In Fig. 3, it can be verified that the codes of the MQI admit interpretation in the CDSC. The main limitation that we observe in the MQI refers to the epistemic dimension when considering that the two standards "Richness and development of mathematics" and "Language" do not reflect with the necessary detail the complexity of the mathematical knowledge that is intended to teach.

CONCLUSIONS:

There is a radical difference in the purpose pursued with developing the quality and suitability constructs. The quality of instruction has been thought as an instrument to "measure" the characteristics that the mathematics teacher work should have in order to help in the processes of teacher selection. On the contrary, the construct suitability has been thought as a tool that allows the teacher to reflect on its own practice and to guide its improvement in the context where the teaching and learning process takes place.

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