Ecology of Mathematical Knowledge: An Alternative Vision of the Popularization of Mathematics

by Juan D. Godino
University of Granada, Granda (Spain)

The title of this round table, *Let's Cultivate Mathematics!*, leads us in a natural way to the use of an implicit metaphor: mathematics can be considered to be like a plant that grows and develops in certain habitats and which needs care to maintain its vitality. We believe that the analysis of the problem of the use of mathematics in different institutions and their relation to other sciences can be facilitated by viewing such a problem in the light of the ecological paradigm, i.e., "the scientific discipline that is interested in the relations between the organisms and their past, present and future environments. These relations include the individual's physiological responses, the structure and dynamic of the populations, the interactions between species, the organization of the biological communities and the processing of energy and matter in the ecosystems " (*Ecology*, Journal of the American Ecological Society).

Since Lakoff and Johnson's work (1980) on the relevant role of metaphorical concepts in structuring the human conceptual system, the use of metaphors has been justified as a means to understand and experiment one reality in terms of another. We think that the ecological metaphor proposed by Chevallard (1989) in didactic analysis constitutes an important tool in order to understand the genesis, the development and the functions of mathematical knowledge in human institutions. The analysis of the institutional ecology of a piece of knowledge leads us to study its habitats, i.e., the places where we can find the objects with which it is associated, the supporting structures and the functions of these interrelations, that is, the ecological niches of the different aspects of mathematical knowledge.

In this presentation we are interested in the problem of the use of mathematics, its characteristics and conditions of development in present-day culture and society, making use of the ecological metaphor and, to be more precise, of the concept of econiche. A modern approach to this concept, based on the general theory of systems (Patten and Auble, 1980) enables us to apply it to inanimate objects, by replacing the criteria of viability, persistence or indefinite existence, by the notions of utility, availability or fitting.

Ecology of Mathematical Knowledge

The application of the ecological metaphor to the study of the evolution of knowledge implies that we consider it as a system of "organisms" or "objects" that interact and play a role within the institutions where its cultural existence is recog-

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nised and which have become its habitat. It seems clear that it is not possible to think of knowledge regardless of the people who think and use it. However the identification of the existence of a piece of knowledge needs collective recognition. Institutions are thus the habitat of knowledge (Berger and Luckmann, 1968). One of the possibilities that the ecological paradigm offers consists in its capacity to give meaning to new questions that otherwise would seem obvious or uninteresting. Likewise, it leads to focus our attention on contextual aspects and interactions that frequently go by unnoticed. As an example of this we indicate some of these questions.

(a) Which institutional habitats does mathematical knowledge occupy at present? What are the different uses of mathematics in these habitats?

(b) Are there any institutions in which mathematics could be used more intensely and adequately?

(c) What types of environmental constraints (limiting factors) make it difficult for mathematics to occupy empty ecological niches?

(d) How are mathematics related to the mathematical knowledge existing in different institutions?

(e) Is it possible to identify subspecies (subknowledge) as a result of the phenomenon of adapting to the environment?

(f) Are there any special relations of competition, symbiosis and dominance and control between knowledge and subknowledge that condition the diffusion of mathematics?

(g) In general, in society, mathematics is not sufficiently appreciated and so leads a precarious existence: What are the factors that determine this "mathophobia"?

We will try to offer some partial answers to these questions. It is interesting to identify, in the first place, three types of mathematical "subspecies", as a result of adaptation to different institutions, i.e., pure, applied and school mathematics. The coexistence between these sub-species is not free of problems which make an optimal diffusion of mathematics difficult. The didactic transposition (Chevallard 1985) can be regarded as the phenomenon of adapting pure and applied mathematics to the school environment. Also between the latter two we would have to differentiate phenomena of mutual adaptations that we could call "modelling transpositions". The subspecies sometimes coexist in the same institutions. At the university level, for example, the normal thing is for teachers to be researchers involved, at the same time, in the production of new knowledge. A problematic aspect is the phenomenon of "dominance and control" which the PM institution (pure mathematics) exercises on AM (applied mathematics) and EM (educational mathematics), which has negative consequences. Applied mathematics is frequently seen by "pure mathematicians" as something of an inferior category. The doses of creativity required in the process of modelling real-world problems and in the necessary educational contextualization is frequently ignored and underestimated.

Within the different habitats of mathematics, the first in order of importance with respect to extension is Mathematics Education, understood as a system that includes not only the teaching and learning of mathematics but also curricular design, the elaboration of didactic material, and didactic research. At the different educational and professional specializations levels, mathematics is found to be omnipresent in education, although the phenomena of dominance and control of theory over applications, to which we have referred, is reproduced. But, regardless of the tool-object dialectic that is characteristic of mathematics (Douady, 1986), mathematics education frequently emphasises the conceptual aspects (objects) as opposed to the applications.

In teaching as well as in research or the applications, mathematics coexist and interact with other knowledge, giving rise to new fields or "species", as for example econometry, psychometry, etc. (Benzecri, 1982). New problems of competition of the sciences over mathematics, i.e., usually professional mathematics, arise. Another conflicting aspect is the difficulty of communication between the mathematician and the specialist in other sciences, due to the use of scientific languages that are different in each specialty. The users of mathematics are those who pose the problems, but it is the mathematician who has the tools to solve them. In the process of posing the problem to the mathematician by the user and of communication of the solutions achieved by the mathematician, a double process of didactic transposition appears from one discipline to another. In this process there can be disagreements that perturb the adequate use of mathematical tools.

It is expected on other occasions that the response to a mathematical problem is immediate, without any sort of creative reflection (Barnett 1988). In school practice, each problem has a solution, which is frequently unique, and in every case the teacher knows this solution. Society does not appreciate the professional mathematician because it is understood that mathematics teaching, from school to university, should make citizens and different professionals capable of solving their own mathematical problems. This is unreal and prevents an adequate cultivation of mathematics. There are usually different mathematical techniques adapted to a given problem. Moreover, each one of them is based on a series of hypotheses of a theoretical nature which are really satisfied. The professional mathematician should assess, among the available methods, the degree of fitness between hypothesis and data. Mathematical modelling is frequently highly complex and requires sophisticated technical skills as well as a level of creativity. This can only be achieved by people with a certain level of specialization and professional dedication.

On the other hand, the application of many routine mathematical techniques is today a laborious task, due to the great quantity of information that it is necessary to process in these applications. The computer is necessary. This poses the problem of the cooperation with yet another science, i.e., computer science. The mutual influence between both disciplines is full of potential but also of difficulties.

Problems of the Popularization of Mathematics

Recently the term popularization has been coined to refer to a set of activities whose specific objective is to try to fill the gap between science and the public's understanding of the same (Howson and Kahane, 1990). It deals with trying to share mathematics with the widest possible public, to encourage people to be "mathematically active", to induce a recognition and a favourable attitude towards mathematical activity. The aforementioned authors distinguish the concept of popularization from that of mathematics teaching by its specific characteristics:

— it deals with freely made decisions that are not subject to the compulsory character of teaching: they do not suppose hard work, but freedom and pleasure;
— it is proposed for a wider audience; it should affect all groups of people, from the researchers themselves to retired people;
— it tries to use all modalities of communication;
— it affects all mathematical topics.

The necessity of carrying out activities of popularization arises from the present reality of rejection, reluctance and the bad image that mathematics has among the public in general. "The popular image of mathematics is that it is difficult, cold, ultra-rational, important and strongly masculine" (Ernest, 1992). This situation is negative, from a personal as well as from a collective point of view. In every country there is a need to increase the number of scientific professionals and a bad image of mathematics implies a handicap in this respect.

The activities of popularization are interpretable in the perspective of the ecology of knowledge, as efforts to use mathematics in a wide variety of institutions, and to a certain extent in a way adapted to them. So, it has to do with the promotion of the use of mathematics in all the potential ecological niches, by using the necessary selection and adaptations. To each group should be proposed some mathematical activities adequate to their interests and possibilities. This implies recognition of a greater variety of types of relations to mathematical knowledge, constructed by the subjects themselves by using a very wide range of mathematical activities. The popularization thus arises as an emblem of a new epistemological formation, of a new "knowledge" that competes to a certain extent for the same space as the didactics of mathematics.

In general terms the objectives of popularization seem valuable and necessary. But the ecological perspective leads us, before proposing indiscriminated actions, to reflect on the "biotic" and "abiotic" factors that determine mathophobia. The heart of the problem that the popularization is trying to remedy, i.e., the social rejection of mathematics, cannot be solved with the type of activities that are currently proposed. To understand mathematics is something so complex, that includes so many nuances and levels (Sierpinska, 1989), that it seems ingenuous to try to achieve it merely with the activities of popularization. The analysis of the conditions on which a favourable attitude towards mathematics depends, is the object of study of one scientific discipline: the didactics of mathematics. The difficulty to fulfill these objectives is obvious if we observe the embryonic state in which this discipline is found in a majority of countries.

It seems clear that the aforesaid failure should be attributed to the particular type of existence that the "educational noosphere" (Chevallard, 1985) has given to date to school mathematics, through the processes of didactic transposition. However, the actions that are proposed as a counterpoint to achieve the objective of popularization do not seem clear. To learn to see the mathematics crystallized in products, to illustrate textbooks, to propose passtimes and crosswords in newspapers, is this enough to make mathematics activity accessible and pleasant for the many?

In our opinion, the actions of popularization do not always have the adequate orientation. It is not always possible to make mathematics pleasant since they do not necessarily have to be so. It is not a pleasant task to transpose a matrix or develop a function in power series. We must create situations in which the use of these objects is the most reasonable. It is not necessary to make a hammer popular for someone to use it to put in a nail. Nobody likes to have a vaccination; however, responsible mothers periodically have their children vaccinated because they know that medical science has demonstrated the efficiency of these vaccinations in the face of illnesses with serious consequences. In the case of medicine, the popularization of the same has not consisted in making it pleasant, but in putting it within the reach of everyone through health centres and professional doctors. For mathematics, the key problem lies in the fact that the pupil (the citizen) receives complex, incomprehensible and, accordingly, useless knowledge at school.

Some Proposals for Action

The teaching institutions (schools, universities, etc.) can be seen in this perspective as special habitats of knowledge, as "nurseries" where these "organisms" grow, since the users of mathematics in the different fields of society receive their education in these centres. Mathematics should be contextualized, adapted to the conditions of particular habitats. The contrary would generate risks of a general rejection.

The development of a common language which makes mutual understanding and communication possible is necessary for the development of a "symbiotic coexistence". This requires several types of actions among which we point out:

— A formation of all the citizens and professionals relating mathematics to the problems of their own environment and interests, and enabling them to distinguish the situations in which the collaboration of the expert mathematician is necessary. Mathematical instruction should provide each citizen with the capacity to identify mathematical situations and to discern when the collaboration of a mathematical professional proves necessary.

— The creation of mathematics consultancies in the universities, in symbiosis with the computer consultancies, and perhaps in the secondary school institutions, that permits the creation of habits that facilitate the integration and the cooperative use of different fields of knowledge.

— The setting up of interdisciplinary teams in research and development units is likewise a key action for mathematics to be used intensely and adequately.

Finally we consider that it is fundamental to support the development of didactic studies, since these analyze and identify the conditions of the "support structures" (Alley, 1985) of these "organisms" in the different institutions where they can survive. Didacticians, the group of people who critically and systematically reflect on the production and communication of knowledge, play the role of "fertilizers" for the knowledge to fully develop its potential.

References

[1] Alley, Th. R., "Organism-environment mutuality epistemics, and the concept of an ecological niche", *Synthesis* **65** (1985), 411–444.

[2] Alsina, C. *et al.*, Hacia unas matemáticas populares, *Suma* **4** (1989), 83–120.

[3] Barnet, V., "Statistical consultancy. A basis for teaching and research". In: *Proceedings of the Second International Conference on the Teaching of Statistics*, R. Davidson and J. Swift (eds.), University of Victoria, 1988, pp 303–307.

[4] Benzécry, J.-P., *Histoire et préhistoire de l'analyse des données*, Dunod, Paris, 1982.

[5] Berger, P. and Luckmann, T., *La construcción social de la realidad*, Amorrortu, Buenos Aires, 1968.

[6] Chevallard, Y., *La transposition didactique*, La pensée sauvage, Grenoble, 1985.

[7] Chevallard, Y., "Le concept de rapport au savoir. Rapport personnel, rapport institutionnel, rapport officiel", Séminaire de Didactique des Mathématiques et de l'Informatique, Université Joseph Fourier-Grenoble I, 1989.

[8] Douady, R., Jeux de cadres et dialectique outil-objet, *Recherches en Didactiques des Mathématiques* **7**: 2 (1986), 5–31.

[9] Ernest, P., "The Popular Image of Mathematics", *Philosophy of Mathematics Education Newsletter*, 4 & 5, 1992, p.19.

[10] Howson, A. G. and Kahane, J. P., *The Popularization of Mathematics: A study overview*, ICMI Study Series, Cambridge University Press, 1990.

[11] Lakoff, G. and Johnson, M., *Metaphors We Live By*, University of Chicago, Chicago, 1980.

[12] Patten, B.C. and Auble, G. T., "Systems approach to the concept of niche", *Synthesis* **43** (1980), 155–181.

[13] Sierpinska, A., "Some remarks on understanding in mathematics", *For the Learning of Mathematics* **10**:3 (1990), 24–36.