EVALUATION OF THE EFFICIENCY OF OUR TEACHING PRACTICE BASED ON OSA THEORY: THE CASE OF LINEAR ALGEBRA COURSE IN B-LEARNING

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Abstract

There is presented some experiences in Undergraduate Education at the Morelos State University (México) concerned to the new educational model (Blended learning) which provides flexibility in learning processes and formation of subjects. Our educational innovations have been developed in the frames of the research project “Implementation of the innovation strategies in order to attend the educational programs in Mathematics for different terminal areas at the Faculty of Sciences, UAEM”.

Keywords: Efficiency of teaching practice, Ontosemiotic Approach, Linear Algebra, B-learning.

1 INTRODUCTION

In the B-learning modality the interaction between teacher and students is reduced to the online communications, leaving aside the so-called metaphoric, gestural, etc. forms which take place in the presence modality.

We consider that in the B-learning modality the traditional curricula of Linear Algebra course can be enhanced by problem-situations, where students can concentrate on the most important concepts (instead of tedious algebraic manipulations) as well as on the corresponding interpretations of geometric and physics nature, so that the visualization of many difficult situations in applications may help to grasp the underlying ideas.

This approach dictates the necessity to design resources to be implemented in the MOODLE platform, which would serve as useful instructions in the process of independent studies.

Analyzing the traditional teaching practice according to [1], it has been detected that there are various complex mathematical objects in the linear algebra course which are not illustrated in the recommended textbooks and, as a consequence, are not explained in traditional teaching instructions.

In our work we rely upon the theory of Ontosemiotic Approach (OSA), which provides a generalized point of view of mathematical objects and their relations in terms of institutional and personal practices. The onto-semiotic approach to mathematical cognition treats the problem of meaning and the representation of knowledge; knowledge is linked to the activity in which the student should be involved. The mathematical activity plays the central role in OSA theory and is modeled in terms of system of operative and discursive practices.

As well, In OSA it is considered that the analysis of educational practice is an important part of the professional development because it permits reflection on the various issues that condition the teaching and learning processes [1]. Due to the complexity of these processes some theoretical tools should be applied to facilitate the evaluation of the efficiency of our own practice.

In the light of this theory we propose some teaching resources which can serve for deeper understanding of basic complex mathematical objects and may help to develop intellectual abilities. In order to achieve the significant mathematical activity some extra-mathematical problems have been designed in order to provide the active engagement of students with the real experience. Also we consider that the special guiding instructions should motivate students and create genuine interest to study and comprehend these complex mathematical objects so that the multiple representations of them in different intra and extra mathematical contexts could be properly distinguished.

1.1 Evaluation of the efficiency of our teaching practice based on OSA theory

To evaluate the pertinence of the mathematical instruction process in teaching Linear Algebra course and to determine guidelines for improving the design of this process and implementation of resources...
in B-learning modality we consider those mathematical practices and interactions which are most appropriate to achieve the main objectives of the course.

Such sort of evaluations to be made properly need some theoretical tools and should be based on some theoretical background, although this does not exclude the possibility of carrying out a partial evaluation of the suitability of a study process.

The principal trajectories which characterize the efficiency of teaching practice according to OSA are treated in [1]. In general, the authors of OSA prefer to select a list of processes that can be considered important in mathematical activity, rather than formalize the definition.

We agree with OSA that mathematical didactics should not be limited to mere description, but should aspire to improving the orchestration and development of study processes [1].

Since there is a need for criteria of suitability or appropriateness that permits evaluation of the improving process carried out and “guide” their improvement, we have found that the theoretical construction in [1] provide such an evaluative perspective.

We focus on interactions of the theoretical ideas and operational practices which lead to the further applications in extra-mathematical contexts.

1.2 Principal themes of Linear Algebra course and innovations for B-learning

1.2.1 Brief analysis of the traditional teaching practice

The standard themes of linear algebra courses concern the notion of n-dimensional vector space (linear space) and linear transformations of such spaces, or, more generally, linear applications between vector spaces. Usually the variety of examples is presented to demonstrate the importance of each requirement posed in the definition.

Traditionally the education in Linear Algebra courses is mostly restricted to operational practice leaving aside many questions that could be successfully treated since the place is just adequate for corresponding situation problems. Also the time for teaching theory is restricted even more according to the new competence focused paradigm.

1.2.2 Towards intra and extra mathematical applications within standard themes of linear algebra courses.

We suggest some preliminary Situations (problems) of extra or intra-mathematical applications on the early stages of the course in order to involve students in the activities of “personal cognitions” and to draw their attention to alternative point of view on the same situation.

We focus on interactions of the theoretical ideas and operational practices which lead to the further applications in extra-mathematical contexts and pay attention to some topics that are important for real competition tasks. There is a possibility provided by MOODLE to arrange some tasks within the so called WebQuests.

2 THEORETICAL GUIDELINES

In our innovations we rely upon the theory of Ontosemiotic Approach (OSA), which provides a generalized point of view of mathematical objects and their relations in terms of institutional and personal practices.

2.1 Some principal postulates of the OSA

The OSA theory allows us to facilitate the students’ learning process in their independent studies, especially in the modality of e-learning, explaining the terms of semiotic function, for example, OSA involves some cognition dualities, in particular, expression-content (semiotic function), which permits to detect so called semiotic conflict due to non-adequate applications of the meanings of similar semiotic registers [2]. In the OSA the Knowledge is considered to be indissoluble linked to the activity in which the subject is implied [1]. Mathematics activity plays a central role and is modelled in terms of system of operative and discursive practices.

With respect to Personal-Institutional duality it is considered that Institutional objects emerge from the systems of shared practice within an institution, while personal objects emerge from the specific
practices of an individual. "Personal cognition" is the result of individual thinking and activity when solving a given class of problem, while "institutional cognition" is the result of a dialog, agreement and regulation within the group of subjects belonging to a community of practices [1].

Thus our series of tasks are supposed to facilitate "institutional cognition" by means of directed instructions to develop deeper level of "Personal cognition" involving the class of problems different from those treated within the traditional course: i.e., problems on another level, in another ambient (mathematical environment), where traditional tools should be applied in the contexts different from those employed in the traditional courses.

2.2 Epistemological and didactic trajectories in the evaluation of pertinence

The epistemological analysis is supposed to be carried out by the teacher and is concerned to organization of the material in accordance to the tradition or by its own will.

As far as Linear Algebra courses is concerned, the epistemological tradition, which most textbooks present since 1853, is due to the theoretic development of matrix theory by Arthur Cayley. Historically, the notion of matrix was proposed by J.J. Sylvester and communicated to A. Cayley sometime before. This mathematical object emerged in the context of linear systems of n equations with n unknowns, where the crucial role was given to the value of some coefficients (coined as "determinants"), which give possibilities to characterize the system as consistent or undetermined and to obtain the explicit solution if this coefficient is different from zero.

Those coefficients are calculated by a very specific rule from the data of the initial linear system (depending also of the order of the system), thus the role of the table whose entries represent these data is rather secondary: nevertheless J.J. Sylvester called this arrangement as "Matrix" meaning the "Mother of the Determinant".

Starting from the foundations of matrix theory elaborated by A. Cayley for the most general cases, the notion of determinant is therefore is defined as the function of any squared matrix (i.e., such table that has the same number of rows and columns). In such a context this concept showed vast applications in almost all branches of mathematics.

2.3 Enhancement of linear algebra courses with tasks of investigation type

In order to reach the conjunction of education with Investigation it is important to be able to solve problems beyond the operational routine:

Analyzing lists of problems in linear algebra course and the results in recent researches in mathematics education we noted that there is a lack of problems which would require creative applications of the methods y technics and also would lead toward applications of the theoretical facts beyond the themes covered in standard courses. We consider that the most valuable would be problems which involve interdisciplinary considerations.

Our objectives confine to implement some tasks of exploration type which require an integral approach to intra mathematical inquires. The tasks with investigation objectives which involve applications, where the fusion of theoretical considerations, analytical methods and operative technics is required.

Thus the value of theoretical constructions and methods taught in standard courses would be appreciated on the junctions with other disciplines.

This practice may facilitate the comprehensions of various topics in future optional courses and to help to develop practice of argumentations which involves various aspects of mathematical apparatus and fusion of mathematical techniques.

3 DESCRIPTION OF THE MAIN SITUATION-PROBLEM

Recent publications in Mathematics Educations demonstrate interest to the processes which permit articulate different parts of mathematics as well as mathematics with the extra mathematics reality, that are related directly to the complexity of the processes of teaching and learning.

The most important part of Linear Algebra course, moreover this is the general objective, is the theme concerned to Liner Transformations (of vector spaces or some geometric spaces), where the matrices which characterize (or describe) any linear transformation appeared to be some tables whose entries are specifically arranged by the components of images of basic elements of the space under
consideration (sometimes the way of this arrangement may produce so-called semiotic conflict: we pay special attention to this point in our guiding instructions).

Our main Problem-Situation is related to the remarkable possibility to express any linear transformation, represented by some square matrix, as a sum of the two linear transformations of special kinds: one represented by symmetric matrices and the other with anti-symmetric ones. Usually this decomposition is treated within routine exercises and is applied simply to matrices without further applications or considerations in some context related to transformations.

The properties of such sort of decomposition in the context of the Situation-Problem is rather easy to understand at the level of students taking the linear algebra course which corresponds usually to the 1st year of Higher Education, because it is not so difficult to visualize the small element of some sort of fluid which is moving along the stream, and simultaneously undergoing some displacement, rotation and also some expansion or construction due to the forces interacting in the process.

The students are supposed to relate antisymmetric component of the matrix, which represent the total transformation, with the rotational motion (moreover, to be able to associate the well-known vector product operation to this rotation).

For the symmetric component there should be applied the standard process of diagonalization (the main goal of any course of Linear Algebra) followed by the interpretation of the diagonal entries of such a matrix. These elements just give the values (coefficients) of expansion (contraction) along the new axis of coordinates constructed in the process of diagonalization.

Thus the students encountered the real problem to understand the meaning of another semiotic representation of the same initial object.

This is crucial because it is related to the general problem of recognition when two semiotic representations are related to the same non ostensive mathematical object, in our case it is related to the representations of linear transformations by matrices assigned in different referential systems.

We consider that all traditional operational practices needed to accomplish this problem-situation should be realized beforehand: so that student should only recognize the situation suitable to apply the previous knowledge in another environment.

4 AUXILIARY RECURSES

To accomplish with auxiliary resources for the main Situation-Problem we arrange the preliminary practices within several WebQuests, which is known to be the most appropriated tool provided by the platform MOODLE in Blended Learning modality: one of them just concerns to the characteristics of linear transformations corresponding to symmetric matrix and to the antisymmetric one, which is most important in applications to mechanics, etc.

4.1 WebQuests

Within the theme of vector spaces, as has been described above, some remarkable subsets of vector space of squared matrices is considered as illustrative examples: the set of symmetric matrices and the set of skew symmetric ones, just to emphasize that each subset form a vector space, both being vector subspaces. Students are only asked to find the corresponding dimensions by constructing corresponding bases.

But it is really a surprising discovery for the students that any matrix can be presented as a sum of two matrices: one is the symmetric and the other is skew symmetric.

Such examples in textbooks are rarely related to further applications of this phenomenon: the very important one, which is pertinent to mention here, leads to the notion of direct sum of vector spaces, crucial notions for the mathematics as a whole.

4.1.1 Preliminary explorations

Another blog of situations-problems is concerned to the characteristics of linear transformations corresponding to symmetric matrices and to the antisymmetric ones which is most important in applications to mechanics as we see in our main Problem-Situation.

Next theme, more abstract, is about linear maps (applications) between two vector spaces of dimensions m and n. Here, in the presence of corresponding referential systems in each space, the
one to one correspondence between the set of all linear maps (applications) with the usual linear operations and the vector space of mxn matrices is established, which permit to deduce that the vector space of linear maps (applications) has dimension equal to the product of m by n.

As the goal student may apply the fundamental theorem which states that any two finite dimensional vector spaces can be considered as the same object: say, arithmetic space of real n-tuples.

This permits to achieve that students develop their abilities in a new environment and to demonstrate the variety of applications of mathematical notions.

Also we suggest a blog of exercises for applications: matrices can be associated to the equations which may represent the variety of superificies in tridimensional geometric space used in architecture constructions, to the equations second order of curves in plane, which are conics in the plane: in such cases they are quadratic matrices which turn out to be symmetric.

Among all linear maps in the Linear Algebra course much attention is dedicated to those that are invertible (which requires considerations of dimensions).

Principal interest of invertible linear maps (applications) is when they are realized within the same vector space, in such a case they are called linear transformations.

4.1.2 Discursive practices

Here we arrange the discursive practices concerned to multiple semiotic representations, which are called negotiations of semiotic representations.

Ambient space now is vector space of all matrices, i.e., the set whose elements are all square matrices A with the real numbers as its entries, \( (a^i_j) \), \( i,j = 1,2, \ldots n \).

This set is a linear space with respect to the addition of elements and the multiplication by scalar from the set of real numbers.

But, why do we use two different positions for its indices? Usually the indices indicate the place in the array: one of them gives the number of the row and another number of columns, depending on the convention in the context.

The semiotic registers for magnitudes that involve two sub-indices, as in the very fruitful Matrix Algebra theory, can be encountered in every area of Mathematics.

Because of multiple matrix representations the link between analysis in terms of representations and analysis in terms of onto-semiotic configurations is necessary for obtaining a better understanding of the complexities of the mathematics teaching and learning processes.

4.2 Learning a bit about the Nature’ laws

We intend emphasize by means of other type of explorative type activities, that there is another reason for multiple matrix representations, which has more deeper meaning: The upper index (super index) \( i \) indicates that the law of transformations with respect to this position is the same as for a vector, meanwhile the position of the lower index (sub index) \( j \) indicates that the law of transformations with respect to that position is different and so characterize a new mathematical object which is called co-vector because the new coordinates are obtained from the initial ones by means of different matrices, which are mutually inverse and are obtained from the matrix which relate the two referential systems. In further intra-mathematical context the existence of co-vectors leads the concept of dual space, very important in applications in modern theories on Mechanics and Physics.

Moreover it has direct concern to validity of the laws of Nature, since the laws should not depend of the referential system. Furthermore any fruitful theory should establish the invariants: in our case these are mathematical quantities expressed in terms of the matrices and are of great importance.

Those could be interpreted and visualized according to the context: for example, the sum of diagonal elements is related to the divergence in Mechanics of fluids, and another invariant is the determinant, which can be visualized as the volume in tridimensional setting.
5 TASKS FOR COMPUTER SCIENCE STUDENTS

5.1 Enhancement of Linear Algebra course with applications to computer 3D vision

Different investigations indicate the separation of concepts which are intrinsically related. In the OSA [3] it is considered that the processes, where the mathematical objects emerge as a result of the computational and discursive practices, are very complicated and there should be distinguished at least two levels: first, there emerge primary objects, which are organized as epistemic configurations, the second level depends on different factors, for example, in our case, as practical benefits of mathematical objects employed in Computer 3D Vision. The discursive practice should emphasize the interpretation of mathematical objects as different from their ostensive representations, because the mathematical entities are presented as “objects with properties” which are independent of individuals who encounter and explore these objects.

These theoretical indications can be directly applied to teaching the mathematical background of Computer 3D Vision, which, in essence, is the linear algebra and the corresponding geometric interpretations.

How do these mathematical theories (essentially, Euclidean and Projective geometries) provide an adequate description of a rigid body motion (the camera in this context)? Students should understand clearly through analytic representations that preserving distances between points is not sufficient to characterize a rigid object moving in space: there are transformations that preserve distances, and yet they are not physically realizable (the preserving of orientations expressed analytically through vector-product operation is indispensable). One should visualize the transformations and translate their actions in terms of relations between points, taking into account that points cannot move relative to each other. This obviously leads to the useful conclusion: to describe the Camera motion it is enough to choose only one point on it and a rotation of reference frame attached to this point. In order to see this it is necessary to specify the translation and rotation of the scene relative to a frame chosen (which is relative, of course). Thus what does matter is the relative motion between the scene and the camera.

The complexity associated to the mathematical objects treated in this context requires that there should be paid special attention to distinguish the laws of transformations of points and vectors, keeping also in mind that those are different geometric objects, although they have the same coordinate representations, apparently. This is important also because the rigid-body transformations act differently on points and vectors (position of the camera and its velocity), as well to distinguish the same physical quantity viewed from a different vantage point). The corresponding algebraic representations will be described in the nest subsection.

Furthermore, the property of invertibility and composition of motions should be mathematically characterized in terms of corresponding algebraic notions and its explicit matrix representation useful for computation. Rather striking consequence of the well-known first-order approximations, being applied in the realm of matrices for 3D Vision, relates skew symmetric matrices as “tangent vector” to the rotational part of rigid body movement, which can be expressed though the well-defined mapping which relates these two geometric notions.

5.2 Transformations in Euclidean and Affine plane

Traditional examples to facilitate the comprehension of linear transformations is to construct the simplest motions on plane, so called, transformation groups in Euclidean space, and other types of transformations which do not preserve the shape of the figures, called affine transformations. The combination or superposition of two linear transformations reveals a specific rule, called semi-direct product of corresponding groups, each of them being commutative, while the product is not, in contrast with the case of the direct product of groups.

The result of this new operation is called Transformation pseudo group.

Guiding the computational practice of this task, we suggest to students realize the composition of two transformations expressed in coordinates and express the result in the form similar to the initial one.

Then they should extend this practice to general case, choosing matrix representation
Here students are supposed to have theoretical background for the matrix representations and further they are recommended to compare the corresponding semiotic representations with those used in the literature on 3 Vision: 
\[
\begin{pmatrix}
    a & b & c \\
    d & e & f \\
    g & h & i
\end{pmatrix},
\]
or in the shorter form 
\[
\begin{pmatrix}
    A & \zeta \\
    0 & 1
\end{pmatrix}.
\]

6 FURTHER REMAKS

The evaluation of our own teaching pertinence according to the OSA turns out to be favorable for implementation of new type of resources for the Blended Modality. The series of activities which we have designed for our students require higher level of thinking, where the fusion of theoretical considerations, analytical methods and operative technics is required. These activities serve as a preliminary experience in research before their thesis projects.

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