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THE THEORY OF DIDACTICAL SUITABILITY: NETWORKING A SYSTEM OF DIDACTICS PRINCIPLES FOR MATHEMATICS EDUCATION FROM DIFFERENT THEORETICAL PERSPECTIVES

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The Theory of Didactical suitability, whose main aim is to synthesize and organize a system of components and quality criteria for mathematics instruction processes, is described. These criteria are classified into six facets that characterize such processes: epistemic (mathematical institutional meanings), ecological (socio-professional and curricular context), cognitive (personal meanings), affective (emotional factors), interactional (personal interactions) and mediational (didactical resources). The specific suitability criteria take into account, besides the facets mentioned, as well as the interactions between them, other components and assumptions derived from the onto-semiotic approach to mathematical knowledge and instruction. Didactical principles from other frameworks, such as the Theory of Didactical Situations, Realistic Mathematics Education and curriculum documents are also considered.

INTRODUCTION

Didactics of Mathematics research should provide descriptive and explanatory knowledge on teaching and learning specific mathematical content that allow understand these processes. It should also guide, in duly reasoned way, effective actions on the teaching practice and promote its progressive improvement, which requires instructional theories. In this paper we suggest that the notion of *didactical suitability*, introduced in the Onto-Semiotic Approach (OSA) to mathematical knowledge and instruction (Godino, Batanero and Font, 2007), and the system of empirical indicators that develop this notion, may be the starting point for a theory of mathematical instruction oriented to the progressive improvement of teaching.

The complexity of teaching and learning processes requires being extremely cautious in proposing norms and rules for intervention in educational systems. Certainly there are no recipes on how to teach every mathematical content, but this does not mean that it is impossible to identify certain knowledge that allows taking some optimal local decisions. The following methodological assumptions underlay the Theory of Didactical Suitability (DST): Given some specific circumstances (subjects, resources, constraints, ...), an "expert" in Didactic of Mathematics can reason (relying on principles widely agreed and theoretical results empirically tested) that certain sequences of tasks and ways of interaction in the classroom are preferable to other.

In this paper the notion of didactical suitability, introduced by Godino and colleagues in various papers (Godino, Contreras, & Font, 2006; Godino, Bencomo, Font, &Wilhelmi, 2007; Godino, 2011) is presented, as a first step in building a theory of mathematics instruction progressively oriented to improve the practice of teaching and learning mathematics. The elaboration of

suitability indicators for mathematics instruction is proving to be a useful tool in the training of mathematics teachers, as it is shown in Breda, Font and Lima (2015).

FACETS OF DIDACTICAL KNOWLEDGE

The system of knowledge about the teaching and learning of mathematics produced by educational research can be classified according to the following dimensions or facets:

1. *Epistemic facet:* Intended and implemented institutional meaning for a given mathematical content (problems, procedures, concepts, properties, language, arguments) and its different meanings.
2. *Cognitive facet:* Students' levels of development and understanding, strategies, difficulties and errors as regards the intended content (personal meaning).
3. *Affective facet:* Students' attitudes, emotions, motivations and beliefs regarding the contents and the study process.
4. *Media facet:* Didactic and technological resources available for teaching and possible way to use and distribute these resources over time.
5. *Interactional facet:* Organization of the classroom discourse and the interactions between the teacher and students directed to solve students' difficulties and negotiation of meanings.
6. *Ecological facet:* Relationships of the topic with other topics and with the social, political and economic settings that support and condition the teaching and learning of mathematics.

In the OSA, framework anthropological / socio-cultural assumptions (Chevallard, 1992; Wittgenstein, 1953) are assumed for the epistemic and ecological facets; semiotic assumptions (Radford, 2006; Eco, 1978) are adopted regarding the cognitive and affective facets; and a socio-constructivist perspective (Ernest, 1998; Brousseau, 1997) is assumed for instructional facet (interactional and mediacional). The OSA takes also into account the complexity of the teaching - learning mathematics processes due to the systemic interactions between the various facets and components.

DIDACTICAL SUITABILITY

The *didactical suitability* of an instructional process is defined as the degree to which that process (or part thereof) fulfills certain characteristics being classified as suitable (optimal or appropriate) to get the adaptation between the personal meanings achieved by students (learning) and the intended or implemented institutional meanings (teaching), taking into account the circumstances and available resources (environment). This involves the consistent and systematic articulation of the six facets listed above (Godino, Batanero and Font, 2007), each of which is a component of didactical suitability:

a) *Epistemic suitability* measures the representativeness and interconnections of the implemented meaning as regards a previously intended reference meaning;
b) *Cognitive suitability* is the extent to which the implemented meaning is included in the students' "zone of proximal development", and whether the personal meanings achieved is close to the intended meaning;
c) *Emotional suitability* is related to the students' involvement (interest, motivation …) in the study process;

d) *Media suitability* depends on the availability and adequacy of material and temporal resources in the teaching process;
e) *Interactive suitability* depends on the extent to which the didactical configurations and classroom discourse served to identify and solve semiotic conflicts that appeared along the instructional process; and
f) *Ecological suitability* is the extent to which the teaching process fit the school and society educational process, and took into account other factors influencing the setting in which it was developed.

Figure 1 represents the facets and components of didactical suitability of an intended or planned study process, indicating a basic suitability criterion for each facet. *Representativeness* and connections of the intended or planned meanings in respect of a reference meaning previously established (epistemic suitability); *proximity* and cognitive demand of personal meanings to the institutional meaning (cognitive suitability); students' *involvement* in the study process (affective suitability); *negotiation* of meanings (interactional suitability); *availability* of technical and time resources (mediacional suitability); *adaptation* to the socio-professional environment and connections (ecological suitability).

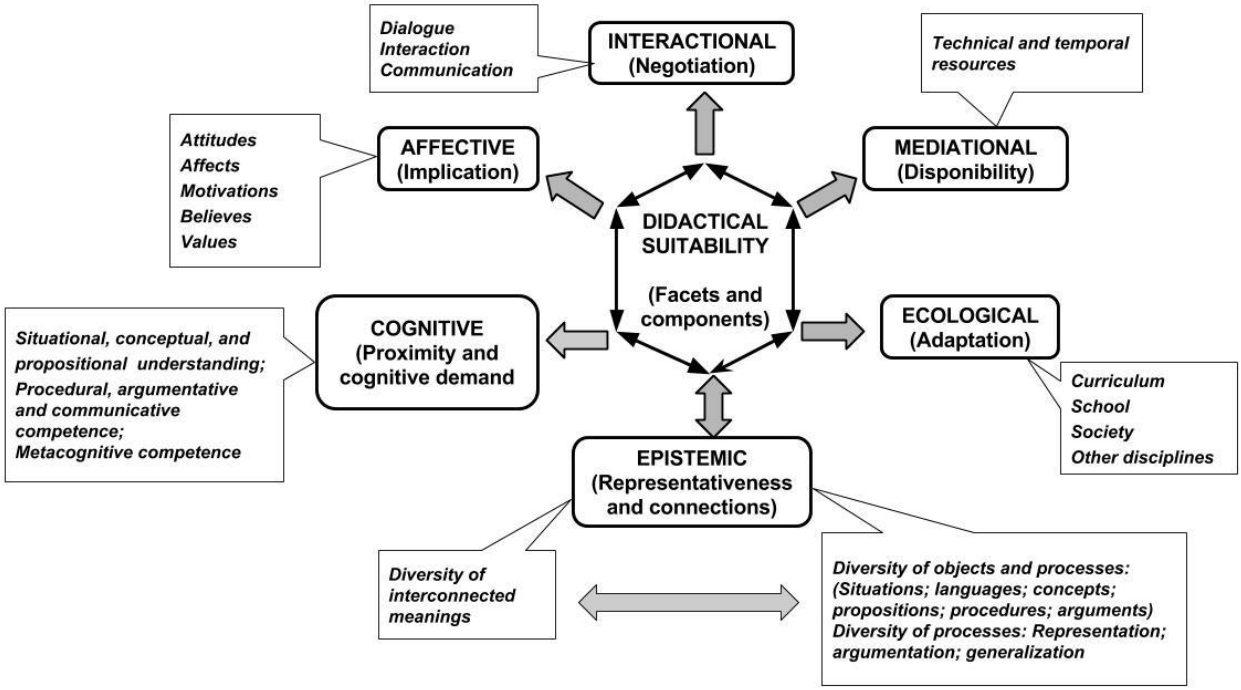

Figure 1: Facets, components and basic didactical suitability criteria

SUITABILITY INDICATORS AND ALIGNMENT WITH OTHER FRAMEWORKS

Starting from previous works (Godino, et al., 2007), Godino (2011) describes a set of empirical suitability indicators for each of the facets and components, and also some concordances between these indicators and others inferred from different theoretical frameworks. For example, in the epistemic suitability a component on situations – problems is included for which the following criteria are stated:

- *A representative and articulated sample of situations-problems that allow contextualize, exercise, apply and generalize the mathematical knowledge, which comes from mathematics itself and other contexts, is presented.*
- *Situations of problem generation are proposed.*

In the OSA theoretical system situations problems have a central role because an anthropological conception of mathematics is assumed; so the emergence of mathematical objects from subject's practices when dealing with certain problems is a key posit. This position is consistent with the "Theory of Didactical Situations" (Brousseau, 1997) and also with the "Realistic Mathematics Education", based on the Didactic Phenomenology (Freudenthal, 1983). Other concordances and complementarities for the various facets are described in Godino (2011), but because space limitations this information cannot be summarized in this paper.

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