Interweaving transmission and inquiry in mathematics and sciences instruction[1]

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Abstract. Despite the huge research efforts that have been made, the problem of how to teach mathematics and sciences remains open. Deciding between teacher-focused teaching models (transmissive teaching) or student-focused (inquiring learning) poses a dilemma for educational practice. In this paper we address this problem and propose a solution applying the Ontosemiotic Approach assumptions and theoretical tools. We argue that the learning optimization and achievement of an appropriate didactic intervention require interweaving in a dialectical and complex way, the teacher's moments of knowledge transmission with the student's inquiry moments. The implementation of efficient didactic trajectories implies the articulation of diverse types of didactic configurations managed through didactical suitability criteria on the teacher´s part. These should take into account the epistemic, cognitive, affective, interactional, mediational and ecological dimensions involved in instructional processes.

Keywords: didactical models, constructivism, objectivism, onto-semiotic approach, didactical suitability

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1. Introduction

Research in mathematics and experimental sciences education is usually interested in describing and understanding teaching and learning processes, avoiding proposing norms on how these processes should be implemented. In research activities the descriptive-explanatory scientific component predominates against the technological component of effective action on educational practice. However, intervention in the real problems of teaching requires developing specific instructional theories that help the teacher to take decisions in the design, implementation and evaluation phases. It is necessary to develop educational theories that articulate the epistemic and ecological facets (curricular theories), together with the cognitive and affective facets (learning theories), oriented to the practice of teaching (instructional design theories). In particular, the optimization of the interactional facet, that is, the types of interactions between teacher and students, between the transmission and inquiry of knowledge, constitutes a problem: The dilemma between directly transmitting knowledge or facilitating the students' inquiry so that they discover and build that knowledge themselves, remains unclear [1].

In this paper, we address the problem of instructional design in mathematics and experimental sciences education from the point of view of the Ontosemiotic Approach to Mathematical Knowledge and Instruction (OSA) [2-3]. We will use the onto-semiotic configuration notion to show the complexity of knowledge, since it allows us to recognize the system of objects and processes put at stake in a problem-solving activity, which constitutes the rationale for such knowledge. Likewise, the notion of didactic configuration helps to recognize the variety and dynamics of teachers and students roles involved in the instructional process of any learning content. The different types of didactic configurations [4] should be articulated forming didactic trajectories whose management by the teacher, have to be guided by suitability criteria [5] in order to achieve the efficiency of the teaching activity. In summary, with the application of the OSA analysis and didactic intervention tools, a theory of instruction has been built for the progressive improvement of mathematics and sciences teaching practice.

In this article we expand and review the invited conference presented at the CISECT [6], incorporating in the thematic thread of the same ideas from previous papers focused on the problem of articulating pedagogical models focused on the teacher or students [7-8], and justifying the relevance of applying a mixed type instructional model. In this sense, an example is described that shows the onto-semiotic complexity of mathematical practices in the case of solving a task on geometric proportionality [9] and another example of an experience with elementary students, following the dialogic-collaborative model described in this article, who have a first encounter with the concept of proportionality [10]. In this article we introduce significant clarifications regarding the type of dialogic-collaborative didactic configuration that we consider suitable in the situations of the student's first encounter with a new content, as well as the relevance of applying this didactic model in the field of education in experimental sciences.

In section 2 we describe in more detail the dilemma between two extreme positions on the types of didactic interactions that should be implemented in instructional processes: constructivism, with an emphasis on student inquiry and autonomy, and objectivism with an emphasis on knowledge transmission. In section 3 we introduce a key factor to consider when deciding between the two extreme positions: recognizing the onto-semiotic complexity of mathematical and scientific knowledge, which must be taken into account, along with other cognitive reasons, in order to ponder constructivism. In section 4 we describe some OSA tools for the analysis and instructional design, which are used in section 5 to present the mixed type instructional model that we propose to optimize the efficiency of didactic activity. This model is explained with an application example in section 6.

2. Constructivism versus objectivism

The family of instructional theories called "Inquiry-Based Education" (IBE), "Inquiry-Based Learning" (IBL), and "Problem-Based Learning" (PBL), postulates inquiry-based learning with little guidance from the teacher [11]. The different varieties of constructivism share, among others, the assumptions that learning is an active process, that knowledge is built instead of innate or passively absorbed and that in order to achieve effective learning it is necessary to approach students with significant, open and challenging problems [12-13].

> "The arguments that human beings are active agents constructing knowledge by themselves have made people believe that instructional activities should encourage learners to construct knowledge through their own participations. This constructivist view plays an important role in science teaching and learning and has become a dominant teaching paradigm." [1, p. 897].

The recommendations for implementing a teaching and learning of mathematics and sciences based on inquiry have been playing a significant role in the curricular orientations of various countries, in projects, research centres and reform initiatives. Linn, Clark and Slotta [14] define inquiry-based science learning as follows:

> "We define inquiry as engaging students in the intentional process of diagnosing problems, criticizing experiments, distinguishing alternatives, planning investigations, revising views, researching conjectures, searching for information, constructing models, debating with peers, communicating to diverse audiences, and forming coherent arguments" [14, p. 518].

In the pedagogical models assuming constructivist principles, the teacher's role is developing a learning environment with which the student interacts autonomously. This means that the teacher has to select some learning tasks and ensure that the student has the cognitive and material resources needed to get involved in solving the problems. In addition, the teacher has to create a cognitive scaffolding, a "choice architecture" that supports and promotes the construction of knowledge by the students themselves. In some way, the aim is implementing a "paternalistic libertarian" pedagogy in the sense of the Thaler and Sunstein [15] "nudge theory", based on the design of interventions of the "nudge" type. "A nudge, as we will use this term, is any aspect of the architecture of choice that modifies the people´s behaviour in a predictable manner without prohibiting any option or significantly changing their economic incentives" [15, p. 6].

In mathematical learning, the use of situations - problems (applications to daily life, other fields of knowledge, or problems internal to the discipline itself) is considered essential, so that students make sense of the conceptual structures that make up Mathematics as a cultural reality. These problems constitute the starting point of mathematical practice, since the problem solving activity, its formulation, communication and justification are considered key in developing the ability to face the solution of non-routine problems. This is the main objective of the "problem solving" tradition [16], whose emphasis is the identification of heuristics and metacognitive strategies. It is also the main aim of other theoretical models such as the Theory of Didactical Situations (TDS) [17], and the Realistic Mathematical Education (RME) [18-19].

There are also positions contrary to constructivism, as is the case of Mayer [20] or Kirschner, Sweller and Clark [21], which justify, through a wide range of investigations, the greater effectiveness of instructional models in which the teacher, and the transmission of knowledge, have a predominant role. These postures are also related to objectivist philosophical positions [22], and to direct instruction or lesson-based pedagogy [23].

[24] state that the last half century empirical research on this problem provides overwhelming and clear evidence that a minimum guide during instruction is significantly less effective and efficient than a guide specifically designed to support the cognitive processes necessary for learning. Similar results are reflected in the meta-analysis performed by Alfieri, Brooks, Aldrich and Tenenbaum [25].

For objectivism, particularly in its behavioural version, knowledge is publicly observable and learning consists of the acquisition of that knowledge through the interaction between stimuli and responses. Frequently, the conditioning form used to achieve desirable verbal behaviours is direct instruction. Cognitive reasons can be provided in favour of applying a didactic model based on the transmission of knowledge (objectivism) versus models based on autonomous construction (constructivism). Kirschner et al. [21] point out that constructivist positions, with minimally guided instruction, contradict the architecture of human cognition and impose a heavy cognitive burden that prevents learning:

> "We are skilful in an area because our long-term memory contains huge amounts of information concerning the area. That information permits us to quickly recognize the characteristics of a situation and indicates to us, often unconsciously, what to do and when to do it" [21, p. 76].

Other reasons contradicting constructivist positions come from cultural psychology. According to Harris [26]:

> "Accounts of cognitive development have often portrayed children as independent scientists who gather first-hand data and form theories about the natural world. I argue that this metaphor is inappropriate for children's cultural learning. In that domain, children are better seen as anthropologists who attend to, engage with, and learn from members of their culture" [26, p. 259].

The metaphor of the child as a natural scientist, so durable and powerful, is useful when used to describe how children make sense of the universal regularities of the natural world, regularities that they can observe themselves, regardless of their cultural environment. However, the metaphor is misleading when used to explain cognitive development. Children are born in a cultural world that mediates their encounters with the physical and biological world. To access this cultural world, children need a socially oriented learning mode (learning through participant observation). "The mastery of normative regularities calls for cultural learning" [26, p. 261].

The debate between direct teaching, linked to objectivist positions on mathematical and scientific knowledge, which defends a central role of the teacher in guiding learning, and a minimally guided teaching, usually referring to the constructivist-type teaching model, is not clearly solved in the research literature. Hmelo-Silver et al. [27] argue that PBL and IBL "are not minimally guided instructional approaches, but provide extensive support and guidance to facilitate student learning" (p. 91). Supporters of problem-based learning and inquiry focus their arguments on the amount of guidance and the situation in which such guidance is provided. They consider that the guide given contains an extensive body of support and being immersed in real-life situations helps students make sense of the scientific content.

For Zang [1], the tension between these two instructional models does not consist of whether one or another would participate in presenting more or less guidance or support to the students, but between explicitly presenting the solutions to the learners or letting them discover these solutions. "For the advocates of direct instruction, explicitly presenting solutions and demonstrating the process to achieve solutions are essential guidance" [1, p. 908]. Having the intention that students discover, explore and find solutions, as structured in IBE, eliminates the need to present such solutions. In constructivist positions, although a certain dose of transmission of information from the teacher to the student is admitted, it is still essential to hide a part of the content. On the contrary for supporters of direct instruction, who assume the theory of cognitive load with emphasis on the worked-examples, providing solutions, is considered essential. In the next section we introduce a new key in the discussion of didactic models based on constructivism (inquiry) and objectivism (transmission). It consists of recognizing the onto-semiotic complexity of mathematical and scientific knowledge [2-28], which must be taken into account in instructional processes intending to achieve the objective of optimizing student learning. By accepting anthropological, semiotic and pragmatic assumptions about mathematical knowledge, it is concluded that an essential part of the knowledge that students have to learn are the conceptual, propositional and procedural rules, agreed within the mathematical or scientific community of practices. To solve the problems that constitute the educational objective, students use their previous knowledge, a central part of which are rules, which must be available to understand and address the task. Intending students to discover those rules is nonsense, but also the objective is to find the solutions, which in turn are rules, and which must be part of their cognitive heritage to solve new problems. The assumptions of an educational-instructional model that would solve the dilemma between inquiry and transmission are obtained by taking into account the onto-semiotic complexity of mathematical and scientific knowledge, while recognizing the central role of problem solving as a rationale for the contents.

3. Onto-semiotic complexity of mathematical knowledge

The onto-semiotic, epistemological and cognitive assumptions of the OSA [2] serve as the basis for an educational-instructional proposal. Although this modelling of knowledge has been developed and applied for the case of mathematics, it is also relevant for the central core (concepts and principles) of scientific knowledge.

The OSA recognizes a key role in the transmission of knowledge (contextualized and meaningful for the student) in the mathematics teaching and learning processes although problem solving and inquiring also have an important part in the learning process. Instruction has to take into account the cultural/regulatory nature of the mathematical objects involved in the mathematical practices, whose competent realization by the students is intended. This competence cannot be considered as acquired if it is meaningless to the students and, therefore, it should be intelligible and relevant to them. Thus, students should be able to use mathematical objects in their own contexts with autonomy. But, according to OSA, due to the onto-semiotic complexity of mathematical knowledge, this autonomy should not necessarily be acquired in the first encounter with the object or in the determination of some of the senses attributed to it; for example, it can be achieved in a mathematical application practice.

How to learn something depends on what you have to learn. According to the OSA the student must make the institutional mathematical practices and the objects and processes involved in the resolution of situations-problems whose learning is intended, appropriate (Figure 1).

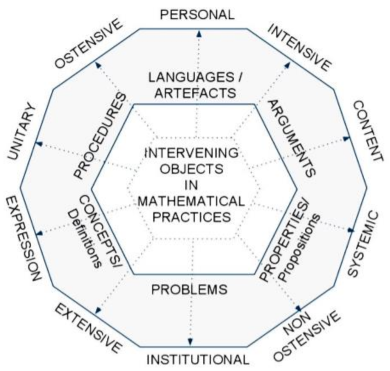


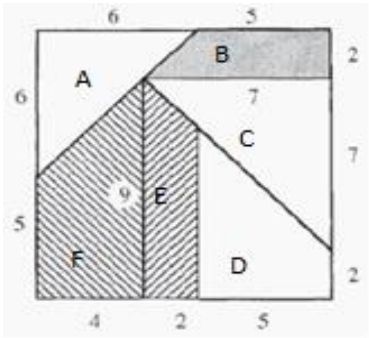
Fig. 1. Onto-semiotic configuration [18, p. 117]

An essential component of these practices are conceptual, propositional, procedural and argumentative objects whose nature is normative [28], and which have emerged in a historical and cultural process oriented towards generalization, formalization and maximizing the efficiency of mathematical work. It does not seem necessary or possible that students discover autonomously the cultural conventions that ultimately determine these objects.

In an instructional process, the student's realization of mathematical practices linked to the solution of some problematic tasks puts into play a conglomerate of objects and processes whose nature, from an institutional point of view, is essentially normative [28]. In the OSA mathematical ontology, according to Wittgenstein's philosophy of mathematics [29-30-31-32], the concepts, propositions and procedures are conceived as grammatical rules of the languages used to describe our worlds. They neither describe properties of objects that have some kind of existence independent of the people who build or invent them, nor of the languages by which they are expressed. From this perspective, mathematical truth is nothing more than an agreement with the result of following a rule that is part of a language game that is put into operation in certain social practices. It is not an agreement of arbitrary opinions, it is an accord of practices subject to rules.

The realization of the mathematical practices involves the intervention of previous objects to understand the demands of the situation - problem and to be able to implement a starting strategy. Such objects, their rules and conditions of application, must be available in the subject's working memory. Although it is possible to individually seek such knowledge in the workspace, there is not always enough time or the student does not succeed in finding that knowledge. Therefore, the teacher and classmates provide invaluable support to avoid frustration and abandonment.

Next, we exemplify the use of the ontosemiotic configuration tool for the case of the proportionality concept, contextualized with the puzzle enlargement task (Figure 2). It is intended to reveal the learning complexity of this mathematical object, discussing the pertinence of addressing such learning globally through a constructivist didactic model, or with a model based on the transmission of decontextualized and meaningless information for the student.



- *You must cut a puzzle similar to this one (the model) on a card. But you have to make it bigger for kindergarten children. This side measuring 4 cm in the model should measure 7 cm in the enlarged puzzle.*
- *You have to be able to make the same figures with the big puzzle as with the model.*
- *To make the big puzzle you will divide into groups. Each group will make a single piece and we will put them all together at the end to fit.*

Fig. 2. Puzzle enlargement situation [17, p. 177]

In the puzzle enlargement situation (Fig. 2) the teacher tries to help the children reach a solution that involves the recognition and calculation of the proportionality constant (scale factor or unit value) through essays and discussions. However, the learning of proportionality requires that students progressively understand the algebraic-functional meaning, as indicated in the following sequence of practices:

– We intend to build a puzzle equal to that of the figure but bigger. That is, if a segment in the model is the union of two others, the associated segment will also be the union of the corresponding ones in the new figure. In addition, if the length of a segment s in the model is multiplied by a number, the length of segment S corresponding to s will be multiplied by the same number.
– Therefore, the correspondence established between the distances of the segments in the model (M) and the distances of the segments in the real puzzle (P), f: M → P, is linear, $f(x) = kx$.
– The coefficient k of the linear function is the proportionality constant in the case of direct proportionality relations between magnitudes.
– Applying the properties of the linear function: $k = k \cdot 1 = f(1)$, and in our case: $f(4) = 7$; $4f(1) = 7$; $f(1) = 7/4 = 1.75$.
– The length of a segment of length x in the model will therefore be $f(x) = 1.75x$ cm in the bigger puzzle.

This sequence of operative and discursive practices put at stake in an algebraic - functional solution involves a system of mathematical objects (Table 1) whose nature is essentially normative and that are the result of a long process of elaboration within the community of mathematical practices.

Table 1. Mathematical objects involved in the algebraic-functional solution of the puzzle situation

Object types	Objects
Languages	– *Symbolic*: function as correspondence between two numerical sets (f: M → P); value of a function f at a point x ($f(x)$); linear function of proportionality

	constant k ($f(x) = kx$). – *Numeric*: fractions, decimals. – *Natural-mathematical*: correspondence, linear function, coefficient, segment, distance, multiplication, union, direct proportionality, magnitude
Concepts	– Magnitude; quantity; measure; numerical value of the measurements; sum of quantities, product by a scalar. – Unlimited sequence of quantities and numbers; functional correspondence; direct proportionality; proportionality coefficient.
Procedures	– Translation of expressions from natural to symbolic language. – Calculation of the proportionality coefficient based on the definition conditions of the linear function. – Calculation of the missing value based on the definition conditions of the linear function.
Propositions	– The correspondence f: M → P is a linear function.
Arguments	– Pragmatic conventions. – The correspondence between the measurements of the two puzzles is additive and homogeneous.

It does not seem pertinent to claim that the student autonomously reconstructs this network of knowledge that the mathematical culture has selected as adequate to respond to proportionality situations. Based on this ontosemiotic complexity, an instructional model based on the presentation of concepts, propositions, procedures and arguments cannot be considered pertinent if this information does not make sense to students.

4. OSA tools for didactical analysis instructional design

In Godino et al. [4] some theoretical tools for the analysis of mathematical instruction processes are developed, by taking into account the previously developed onto-semiotic model for mathematical knowledge. In particular, the notions of didactic configuration and didactic suitability, serve as a basis to define a mixed didactic model that articulates the processes of inquiry and transmission of knowledge, related in a dialectical way in different types of didactic configurations.

4.1. Didactic configuration

A *didactic configuration* (Figure 3) is any segment of didactic activity put into play when approaching the study of a problem, concept, procedure or proposition, as a part of the instruction process of a topic, which requires the implementation of a didactical trajectory (articulated sequence of didactic configurations). It implies, therefore, taking into account the teacher's and student's roles, the resources used and the interactions with the context. In fact, there are different types of didactic configurations, depending on the interaction patterns, and the management of the institutionalization and personalization of knowledge.

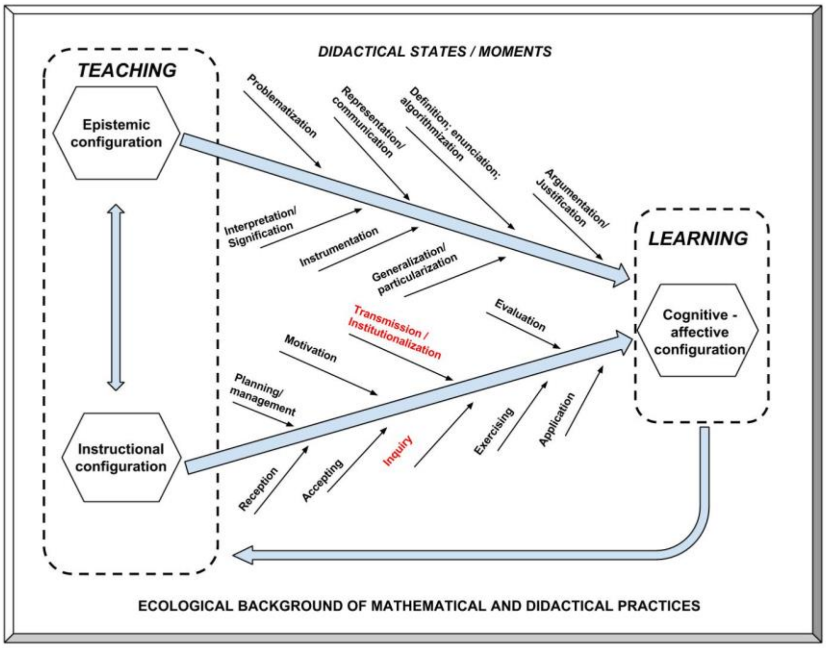


Fig. 3. Components and internal dynamic of a didactic configuration [33, p. 2646]

The task, which defines a didactic configuration, can be formed by different subtasks each of which can be considered as a sub-configuration. In any didactic configuration there is: a) an *epistemic configuration* (system of institutional mathematical practices, objects and processes, required in the task), b) an *instructional configuration* (system of teacher/learner functions and instructional means which are used in addition to the interaction between the different components) and c) a *cognitive-affective configuration* (system of personal mathematical practices, objects and processes that describe the learning and the affective components which accompany it).

Figure 3 summarizes the components and internal dynamic of the didactic configuration, the relations between teaching and learning and the main mathematics processes lined to the onto-semiotic modelling of mathematics knowledge. This modelling takes into account the complexity of the relations that are established in the centre of any didactic configuration, not reducible to moments of inquiry or transmission of knowledge. In Figure 3, with the bottom arrow, from learning to teaching, we want to point out that the relations are not linear but cyclical. In one particular moment of investigation, for example, the learner interacts with the epistemic configuration without the intervention of the teacher (or with less influence). This interaction conditions the teachers' interventions and so, should be taken into account in the instructional configuration, perhaps not totally in its content, but yes however in its nature, need and use. This is obviously not prerogative of the inquiry moments. The cognitive trajectory produces examples, meanings, arguments, etc., which condition the study process and as a result, the epistemic and instructional configurations, thus making possible and committing – in all cases, conditioning- , the conclusion of the instructional project planned.

4.2. Didactical suitability

The detailed analysis of a process of mathematics study, which allows us to reveal the dialectic and synergy between the different components of the didactic system, requires to be divided into units where the notion of configuration and sub-configuration is useful. A *didactic fact* is significant if the actions or didactical practices that make it up carry out a function, or admit an interpretation, in terms of the instructional objective intended. The meaningfulness can be understood from the point of view of the teacher, of the student, or else from an institutional point of view which is external to the didactic system. The notion of didactic suitability [2-5], their facets and components, provide criteria to delimit the relevance of the didactic facts that occur in the processes of mathematics studies.

Didactic suitability of a process of instruction is defined as the degree to which the said process (or part of the same) meets certain characteristics that enable us to say it is optimum or adequate to be adapted among the personal meanings achieved by the students (learning) and the institutional meanings intended or implemented (teaching), taking into account the circumstances and the resources available (context). This supposes the coherent articulation of six facets or dimensions: epistemic, ecological, cognitive, affective, interactional and mediational [2].

5. A mixed transmissive - inquiry instructional model

According to the students' previous knowledge and whether it is a first encounter with the object, or an exercise, application, institutionalization and evaluation moment, the didactic configurations can be of dialogical, collaborative, personal, magisterial, or a combination of these types (Figure 4). The optimization of the learning process through the didactic trajectories may involve a combination of different types of didactic configurations. This optimization, that is, the realization of a suitable didactic activity, has a strongly local character, so that the didactic models, either student-focused (constructivist), or teacher-focused and content (objectivist), are partial visions that drastically reduce the complexity of the educational-instructional process.

In the student's first encounter with a specific meaning of an object, a dialogic - collaborative configuration, where the teacher and students work together to solve problems that put knowledge O at stake in a critical way can optimize learning. The first encounter should therefore be supported by an expert intervention by the teacher, so that the teaching-learning process could thus achieve greater epistemic and ecological suitability [34]. When the rules and the circumstances of application that characterize the object of learning O are understood, it is possible to tend towards higher levels of cognitive and affective suitability, proposing to deepen the study of O (situations of exercising and application), through didactic configurations that progressively attribute greater autonomy to the student (Figure 4).

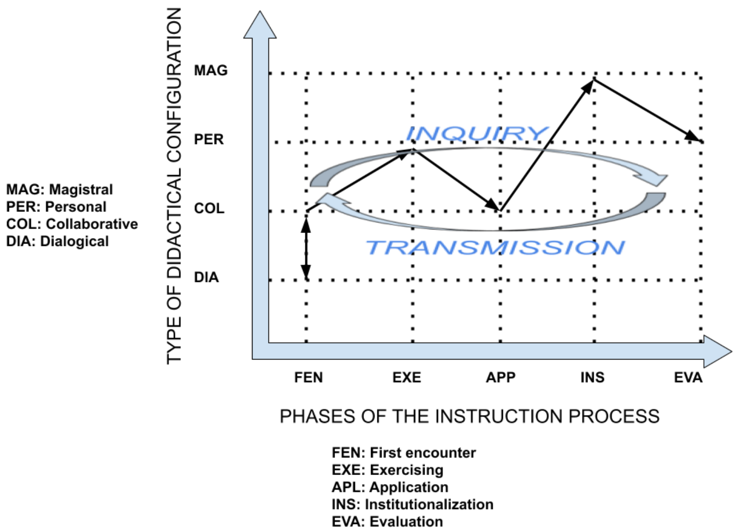


Fig. 4. A mixed inquiry – transmissive instructional model

In the moments or phases of the student's first encounter with a specific meaning of an object, it is considered that a dialogic - collaborative configuration can optimize learning. In these types of configurations, the teacher and students work together to solve problems that put knowledge O at stake in a critical way. The first encounter should therefore be supported by the teacher's expert intervention. The teaching-learning process could thus achieve greater epistemic, ecological and affective suitability [34]. In the phases of the first encounter, through a didactic model with minimal teaching guidance, students are exposed to the risk of not finding any solutions and fall into frustration and task rejection feelings.

> "Even if the students find the solutions on their own, they do not know the most effective procedures as they have to wander around in the problem searching process, not to mention the cognitive loads they are imposed." [1 p. 909].

When the rules and the circumstances of application that characterize the learning object O are understood, it is possible to tend towards higher levels of cognitive and affective suitability, proposing to deepen the study of O (situations of exercising and application), through didactic configurations that progressively attribute greater autonomy to the student in a controlled manner (Figure 4).

In summary, within the OSA framework, it is assumed that the types of didactic configurations that promote learning can vary depending on the types of knowledge sought, the students' initial state of knowledge, the context and circumstances of the instructional process. When it comes to learning new and complex content, the transmission of knowledge at specific times, already by the teacher, and by the leading student within the work teams, can be crucial in the learning process. That transmission can be meaningful when students are participating in the activity and working collaboratively. The didactic configuration tool helps to understand the dynamics and complexity of the interactions between the content, the teacher, students and the context. The optimization of learning can take place locally through a mixed model that articulates the transmission of knowledge, inquiry and collaboration, a model managed by criteria of didactic suitability [2-35] interpreted and adapted to the context by the teacher.

6. Working together introductory situations of proportional reasoning in primary school

Burgos and Godino [10] describe the implementation of the mixed type instructional model described in the previous section, with primary school students, whose aim was to create a first encounter with direct proportionality problems and initiate the development of proportional reasoning in the students. The sample under study consisted of a group of 23 students in fifth grade (10-11 years-old), who had a normal level of performance in mathematics and had difficulties in that course with the issue of fractions, as his tutor reported in an interview.

In a first didactic configuration, students were presented with different everyday situations in which the relationship between quantities of two magnitudes were of direct proportionality. The price paid for different quantities of an item, the distance travelled by a car at constant speed and time. In these situations, two series of numbers appeared, which were represented on the blackboard by means of tables, so that students could recognize the existence of a certain number (the proportionality ratio) that allowed them to write each value of the second series as product of the corresponding values of the first series by the said number. They were also presented with some situations of non-proportionality, in which the students had to decide whether they were or not and why; for example, the age and height of a child. Then the teacher - researcher provided the students with a worksheet with new introductory problem- situations. The students were organized in pairs, following the usual distribution to work in the classroom with their teacher.

In the second didactic configuration focused on the solution of the task "Laura visits her uncle" (Figure 5), it was designed to stimulate inquiry and discussion by means of directed questions that serve as an approach to proportionality. The resolution of this task was carried out in a large group: the students intervene to complete the proportionality table, arguing at each moment the response and discussing with the classmates the strategy followed.

It is the end of the year party and the fifth classes want to order cakes to celebrate it. Laura's uncle is a pastry chef, he makes delicious cakes! So Laura has gone to visit him. That morning he used 3 liters of milk to make 18 equal pies. Laura wants to know how many pies she can make with 6, 2 and 5 liters of milk.

Laura, who is a very smart girl, reasons as follows to form a table like the one shown below.

- First, 6 is double 3 (the number of liters of milk needed for 18 cakes). Put the number of cakes you can make with 6 liters of milk on the table.
- Then she thinks that 2 liters is the third part of 6 liters. Put the following number of cakes on the table.
- Finally 5 liters of milk are 2 liters plus the initial 3 liters.

Finish filling the table following these three ideas.

Liters of milk	*3*	*6*	*2*	*5*
Cakes	*18*			

Can you think of any other way that Laura could complete the table?

Fig. 5. Introductory task: "Laura visits her uncle"

At the end of each activity the ideas were discussed collectively, focusing the attention on the concept of proportionality and the properties whose knowledge and understanding are pursued to develop with the tasks. The students work on the worksheet in a collaborative way and the teacher-researcher could intervene to guide them, remember the necessary information and lead the discussion in the classroom.

One of the tasks that were used to evaluate the learning achieved was Brousseau's puzzle task (Fig.2). 70% of the students correctly solved this task. Miyakawa and Winslow [36, p. 213] affirm, "It appears from experiments done by Brousseau and his colleagues that despite careful preparation in previous lessons, students spontaneously tend to construct the larger pieces by adding 3 cm to all known sides (since 7 cm is 3 cm more than 4 cm)". However, in our study, we found no evidence of this type of strategy, possibly because this task was preceded by other introductory tasks on arithmetic proportionality. The predominant resolution strategy was the reduction to unity, and some students used a resolution procedure that we describe as mixed, since they combine additive strategies with reduction to unity.
In the light of the results achieved, we believe that this model of collaboration between the teacher and the students, regarding the problem-situation that is intended to be solved and the mathematical content put at stake, achieves high levels of suitability in the interactional, cognitive and affective facets. An appropriate degree of dialogue, interaction and communication allowed:

- Detect intuitive, natural strategies and those that students develop with little guidance from the teacher (recurrent use of the tabular register, unit reduction strategy).
- Increase the degree of students' involvement and interest.
- Identify semiotic conflicts (greater difficulty when the proportionality constant is not integer) and resolve them.

6. Final reflections

In this work of research we have complemented the cognitive arguments of Kirschner et al. [21] in favour of models based on the transmission of knowledge with reasons of onto-semiotic nature for the case of mathematical learning and science, especially in the moments of students' "first encounter" with the intended content: what they have to learn are, in a large dose, epistemic/cultural rules, the circumstances of their application and the conditions required for its relevant application. The learners start from known rules (concepts, propositions and procedures) and produce others, which must be shared and compatible with those already established in the mathematical culture. Such rules have to be stored in the subject's long-term memory and put into operation in a timely manner in the short-term memory.

The postulate of constructivist learning with little guidance from the teacher can lead to instructional processes with low cognitive and affective suitability for real subjects, and with low ecological suitability (context adaptation) by not taking into account the onto-semiotic complexity of mathematical knowledge or the potential development zone [37] of the subjects involved.

> "Children cannot discover the properties and regularities of the cultural world via their own independent exploration. They can only do that through interaction and dialogue with others. Children's trust in testimony, their ability to ask questions, their deference toward the use of opaque tools and symbols, and their selection among informants all attest to the fact that nature has prepared them for such cultural learning" [26, p. 267].

We believe that learning optimization implies a dialectical and complex combination between the teacher's roles as an instructor (transmitter) and facilitator (manager) and the student's roles as a knowledge builder and active receiver of meaningful information. The need for this mixed model is reinforced by the need to adapt the educational project to temporary restrictions and the diversity of learning modes and rhythms in large groups of students.

The teaching of mathematics and experimental sciences, should start and focus on the use of situations-problems, as a strategy to make sense of the techniques and theories studied, to propitiate exploratory moments of mathematical activity and develop research skills. However, configurations of mathematical objects (concepts, propositions, procedures, arguments) intervene in mathematical and scientific practice [28], which must be recognized by the teacher to plan their study. Such objects must be progressively dominated by students if we wish they progress towards successive advanced levels of knowledge and competence.

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References

1. Zhang, L. (2016). Is inquiry-based science teaching worth the effort? Some thoughts worth considering. *Science Education, 25*, 897-915.
2. Godino, J. D. Batanero, C. & Font, V. (2007). The onto-semiotic approach to research in mathematics education. *ZDM. The International Journal on Mathematics Education, 39* (1-2), 127-135.
3. Godino, J. D., Batanero, C. & Font, V. (2019). The onto-semiotic approach: implications for the prescriptive character of didactics. *For the Learning of Mathematics, 39* (1), 37- 42.
4. Godino, J. D., Contreras, A. & Font, V. (2006). Análisis de procesos de instrucción basado en el enfoque ontológico-semiótico de la cognición matemática. *Recherches en Didactiques des Mathematiques, 26* (1), 39-88.
5. Godino, J. D. (2013). Indicadores de la idoneidad didáctica de procesos de enseñanza y aprendizaje de las matemáticas. *Cuadernos de Investigación y Formación en Educación Matemática, 11*, 111-132.
6. Godino, J. D. (2019). How to teach mathematics and experimental sciences? Solving the inquiring versus transmission dilemma. In K. O. Villalba-Condori, A. Adúriz-Bravo, F. J. García-Peñalvo & J. Lavonen (Eds.), *Proceeding of the Congreso Internacional Sobre Educación y Tecnología en Ciencias - CISETC 2019* (Arequipa, Perú, December 10-12, 2019) (pp. 71-81). Aachen, Germany: CEUR-WS.org.
7. Godino, J. D., Batanero, C., Cañadas, G. R. & Contreras, J. M. (2016). Linking inquiry and transmission in teaching and learning mathematics and experimental sciences. *Acta Scientiae, 18* (4), 29-47.
8. Godino, J. D., Rivas, H., Burgos, M. & Wilhelmi, M. D. (2018). Analysis of didactical trajectories in teaching and learning mathematics: overcoming extreme objectivist and constructivist positions. *International Electronic Journal of Mathematics Education, 14*(1), 147-161.
9. Godino, J. D., Burgos, M. & Wilhelmi, M. R. (2019). Papel de las situaciones adidácticas en el aprendizaje matemático. Una mirada crítica desde el enfoque ontosemiótico. *Enseñanza de las Ciencias* (in press).

10. Burgos, M. & Godino, J. D. (2019). Trabajando juntos situaciones introductorias de razonamiento proporcional en primaria. Análisis de una experiencia de enseñanza centrada en el profesor, en el estudiante y en el contenido. *Bolema, 33* (63), 389-410.
11. Artigue, M., & Blomhøj, M. (2013). Conceptualizing inquiry-based education in mathematics. *ZDM Mathematics Education 45*, 797–810.
12. Ernest, P. (1994). Varieties of constructivism: Their metaphors, epistemologies, and pedagogical implications. *Hiroshima Journal of Mathematics Education, 2*, 1-14.
13. Fox, R. (2001). Constructivism examined. *Oxford Review of Education, 27* (1), 23-35.
14. Linn, M. C., Clark, D., & Slotta, J. D. (2003). WISE design for knowledge integration. *Science Education, 87*(4), 517–538.
15. Thaler, R. H., & Sunstein, C. R. (2008). *Nudge improving decisions about health, wealth and happiness*. Yale University Press.
16. Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. In: D. Grouws (Ed.). *Handbook of research on mathematics teaching and learning* (p. 334-370). New York: MacMillan.
17. Brousseau, B. (2002). *Theory of didactical situations in mathematics.* Dordrecht: Kluwer A. P.
18. Freudenthal, H. (1973). *Mathematics as an educational task*. Dordrecht: Reidel.
19. Freudenthal, H. (1991). *Revisiting mathematics education. China lectures.* Dordrecht: Kluwer.
20. Mayer, R. E. (2004). Should there be a three-strikes rule against pure discovery learning? *American Psychologist, 59* (1), 14 - 19.
21. Kirschner, P. A., Sweller J., & Clark R. E. (2006). Why minimal guidance during instruction does not work: An analysis of the failure of constructivist, discovery, problem-based, experiential, and inquiry-based teaching. *Educational Psychologist, 41*(2), 75-86.
22. Jonassen, D. H. (1991). Objectivism vs. constructivism: do we need a new philosophical paradigm? *Educacional Technology Research & Development, 39*(3), 5-14.
23. Boghossian, P. (2006). Behaviorism, constructivism, and Socratic pedagogy. *Educational Philosophy and Theory, 38*(6), 713-722.
24. Sweller J., Kirschner P. A., & Clark R. E. (2007). Why minimally guided teaching techniques do not work: A reply to commentaries. *Educational Psychologist, 42*(2), 115-121.
25. Alfieri, L., Brooks P. J., Aldrich, N. J. & Tenenbaum, H. R. (2011). Does discovery-based instruction enhance learning? *Journal of Educational Psychology, 103*(1), 1-18.
26. Harris, P. L (2012). The child as anthropologist. *Infancia y Aprendizaje, 35* (3), 259-277.
27. Hmelo-Silver, C. E., Duncan, R. G., & Chinn, C. A. (2007). Scaffolding and achievement in problem-based and inquiry learning: A response to Kirschner, Sweller, and Clark (2006). *Educational Psychologist, 42*(2), 99–107.
28. Font, V., Godino, J. D. & Gallardo, J. (2013). The emergence of objects from mathematical practices. *Educational Studies in Mathematics, 82*, 97–124.
29. Baker, G. P. & Hacker, P. M. S. (1985). *Wittgenstein. Rules, grammar and necessity. An analytical commentary on the Philosophical Investigations*. Glasgow:

Basil Blackwell.
30. Bloor, D. (1983). Wittgenstein. *A social theory of knowledge.* London: The Macmillan Press.
31. Wittgenstein, L. (1953). *Philosophical investigations.* New York, NY: The MacMillan Company.
32. Wittgenstein, L. (1978). *Remarks on the foundations of mathematics* (3rd. ed.). Oxford, England: Basil Blackwell.
33. Godino J. D., Batanero C., Cañadas G. R., & Contreras, J. M. (2015). Linking inquiry and transmission in teaching and learning mathematics. In K. Krainer & N. Vondrobá (Eds.), *Proceedings of the Ninth Conference of the European Society for Research in Mathematics Education* (CEME9, 4-8 February 2015) (pp. 2642-2648). Prague, Czech Republic: Charles University in Prague, Faculty of Education and ERME.
34. Godino, J. D., Font, V., Wilhelmi, M. R. & Castro, C. de (2009). Aproximación a la dimensión normativa en Didáctica de la Matemática desde un enfoque ontosemiótico. *Enseñanza de las Ciencias, 27*(1), 59–76.
35. Breda, A., Font, V. & Pino-Fan, L.R. (2018). Criterios valorativos y normativos en la Didáctica de las Matemáticas: el caso del constructo idoneidad didáctica. *Bolema, 32* (60), p. 255 - 278.
36. Miyakawa, T. & Winsløw, C. (2009). Didactical designs for students' proportional reasoning: an "open approach" lesson and a "fundamental situation". *Educational Studies in Mathematics, 72*, 199–218
37. Vygotsky, L. S. (1962). *Thought and language*. Cambridge: M.I.T. Press.