An onto-semiotic approach to the analysis of the affective domain in mathematics education[1]

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A subject of growing interest in mathematics education is the affective domain and its effects on the teaching and learning processes, arising different models about its components and conditioning factors. In this paper, we apply the ontological and semiotic categories from the Onto-Semiotic Approach (OSA) to research in mathematics education, to build an inclusive and systematic model to consider affective situations, practices, objects and processes, as well as the corresponding dualities: personal - institutional, ostensive - non-ostensive, extensive - intensive, unitary - systemic, expression - content. The dynamic character of affects (emotions, attitudes, beliefs and values) and their relations with the epistemic, cognitive, interactional and resources is modelled by the didactical configuration and didactical trajectory notions, theoretical tools which include the affective sub-configuration and sub-trajectory as key components. Another result obtained from this work is the revision of the indicators of affective suitability proposed in Godino (2013).

Keywords: affect; onto-semiotic approach; didactical suitability; mathematics education

Introduction

The Onto-Semiotic Approach (OSA) to research in mathematics education (Godino, Batanero, &Font, 2007; Font, Godino, & Gallardo, 2013) suggests a theoretical system of categories to analyse mathematics teaching and learning processes. To describe these processes and to achieve a deep comprehension of the implied phenomena, the OSA considers six facets or dimensions: epistemic, cognitive, affective, interactional, resources, and ecological (Godino, Contreras, & Font, 2006), as well as the interrelations among them. Each one of these facets can be analysed with different detail levels: problems, practices, objects and processes.

Previous works have developed the subcategories required for the epistemic, cognitive and instructional (interactional and resources) dimensions analysis, thus the incorporation of proper models for the affective and ecological facets remains. In this paper, we discuss the affective domain analysis in mathematics education using the OSA categories, extending them at the same time. We will consider the abundant bibliography about affect and mathematical education (Blanco, 2012; DeBellis & Goldin, 2006; Goldin, 2000; Gómez-Chacón, 2000a; McLeod, 1992; Phillipp, 2007).

Although there is no consensus to delimit the different aspects that must be considered when defining the affective domain, we see that most authors agree on including emotions, attitudes, beliefs and values. These are broad categories of affective entities, each of which includes more specific notions. In our case, we are interested in

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the affects related to mathematics, its teaching and learning, or in relation to more specific aspects of the study of mathematics in educational contexts.

The OSA has been applied to the epistemic (institutional knowledge) and cognitive (personal or subjective knowledge) domain, taking as primitive notions the following: situation - problem, practice (mathematics), object (emerging and intervening in practices), meaning (relation between objects) (Font, et al., 2013). These primary entities can be analysed from different points of view (dualities or contextual polarities), giving rise to new types of secondary entities: personal - institutional, ostensive - non-ostensive, extensive - intensive, unitary - systemic, expression - content. Anthropological presuppositions on mathematics are therefore assumed, without ruling out the use of the object metaphor and the adoption of a certain functional (or semiotic) realism on these objects.

Our purpose here is to address the following questions: Is an onto-semiotic approach relevant to the study of the affective domain? Is it possible to bring new insights to affect in mathematics education when tackling it with the OSA theoretical lenses? What theoretical models on affect can be incorporated and aligned with this approach?

The work is organized in the following sections. First, we illustrate the method, based on OSA theoretical tools, such as the notion of meaningful didactical fact (MDF), which will allow us to broaden the ontology and categories of this framework in relation to the affective domain. Primary affective entities (affective situations, affective practices, intervening and emergent objects) and the contextual dualities between them (personal - institutional, expression - content, ostensive - non-ostensive, extensive - intensive, unitary - systemic) are introduced. Subsequently, in the light of this discussion, a review of the empirical criteria of suitability of the affective dimension is suggested, as well as a reflection on the dynamics of affectivity. The different notions will be exemplified by referencing to class episodes experienced by the first author.

Affect and mathematics education

The influence of the affective domain in mathematics teaching and learning is a consolidated research line in mathematics education. However, as Grootenboer and Marshman (2016) notice, along as other authors (Lomas, Grootenboer, & Attard, 2012) there has been a lack of clarity about the nature of affect, as well as the terms used to explain it from the different theoretical frameworks. Anyway, there is some consensus regarding the basic partitioning of the affective domain into emotions, attitudes and beliefs (McLeod, 1992), three interrelated facets whose main difference is the intensity and stability, as well as its relation to cognition. Other authors add other facets in order to provide more suitable models to their research aims. For instance, DeBellis & Goldin (2006) consider values to refer to deep commitments cherished by individuals that help to stablish shorter-term priorities and choices and may also be highly structured. Attard (2014) considers also motivation and engagement.

Anyway, most researchers accept emotions, attitudes, beliefs and values as the key components of the affective domain in mathematics education and use them to study the interactions among cognition, mathematical affect, teaching and learning processes, problem solving, achievement and behaviour (Grootenboer & Marshman, 2016; Pepin & Roesken-Winter, 2015). One of these interactions is the mediation of beliefs in the learning itself. Furinghetti and Pehkonen (2002) suggest that beliefs can be considered also as a form of subjective knowledge and thus they can be interpreted as a nexus between the cognitive and the affective domains. Besides, there are also complex interactions between teacher and student beliefs, particularly in problem-solving approaches and ICT mediation (Depaepe, De Corte, & Verschaffel, 2015; Gómez-Chacón, 2011).

Recently, there has been important advances in the comprehension of affect, and theoretical frameworks on affect have continued its development (Zan, Brown, Evans, & Hannula, 2006; Leder, Pehkonen, & Torner, 2002; Goldin, Roesken, &Toerner, 2009). Our review suggests that there is a clear interest in deepening how the different facets of the affective domain interact among them and with other domains, for what an onto-semiotic approach can be useful, as it incorporates theoretical notions to handle the multiple facets of a mathematics learning and teaching process.

Method

A didactical fact is 'any event that has a place and a time in the becoming of a mathematical instruction process and that, for some reason, is considered as a unit' (Wilhelmi, Font, &Godino, 2005). We will say that it is a meaningful didactic fact (MDF) 'if the didactic actions or practices that compose it play a role, or admit an interpretation, in terms of the intended instructional objective' (Godino, Rivas, Arteaga, Lasa, & Wilhelmi, 2014)

To support the description and understanding of the categories of affective entities, we will apply the notion of MDF to two experiences made in different educational and research contexts. On the one hand, we start from the descriptive and naturalistic research (Hernández, Fernández, & Baptista, 2010; Kelly, Lesh, & Baek, 2008) of an instructional process on probability implemented with a group of 18 students of third year of secondary education (14-15 years-old, from a public secondary education institute in Spain (Beltrán-Pellicer, & Godino, 2017)). The data collection instrument, from which the MDF are extracted, is the classroom journal of the teacher, the first author of this article and who acts as teacher-researcher.

The second experience was also carried out by the first author in a public adult education centre. The students attend secondary education, or some of the courses of preparation for the acquisition of key competences or access to the degrees of vocational training, constituting a diverse sample of 38 people between 18 and 58 years-old. The collected data includes student's productions in their notebooks and a free essay entitled 'Mathematics and I, my relationship with mathematics so far', an instrument used by other authors (Di Martino & Zan, 2011) to analyse attitudes and beliefs.

The selected MDF from both data sources allow us to illustrate specific aspects of the affective domain, as we will see below, through the OSA categories.

Primary affective entities

Following the OSA pragmatic epistemological assumptions, we are now asking for the affective meaning of certain signs (in the sense of Peirce's representamen), in any of the possible registers and representations, which may be verbal or written expressions, observable behaviours, etc. Such meaning must be sought in the systems of practices that a person performs to solve a problem situation, or towards a practice, an object, a mathematical process, or any mathematics study situation.

There is agreement, within the scope of research in mathematical education, that the affective domain consists of three components: emotions, attitudes and beliefs. The origins of this classification go back to McLeod (1992) and, in this article, we will use this ontology of affective objects, to which we will add the values, construct included in the model of DeBellis & Goldin (2006).

Affective situations

It is necessary to consider a specific type of situation that provides the appropriate framework for describing affective practices. When a student is confronted with a situation-problem, an affective situation occurs that juxtaposes itself with the cognitive one, and which comes to include the purely personal meanings about it, in the form of emotions, attitudes, beliefs or values. For example, a mental block emotion towards a kind of problem-situation, a persevering attitude that facilitates the implementation of problem-solving heuristics, or a specific belief about the nature of the mathematical objects involved. In fact, all problem-situations in which the student's active participation is required are strongly affective. Once the situation has been exposed, the personal beliefs of each student come into play, either to mathematics as a subject of study or to the context in which the proposed situation is framed.

However, affective situations do not arise solely in response to a problem-situation, since the teaching and learning ecosystems provide constant reference points for the affective domain. In this way, there are situations of production, communication or, simply, of individual mathematical study. For example, a class session itself can bring up beliefs that influence the student's attitude that day, without the need for any problem-situation yet.

Therefore, it is feasible to describe an affective configuration for each of these situations, which will capture the circumstances of each component of the affective domain: emotions, attitudes, beliefs and values. Since we are interested in the relationships between affect and mathematical learning, we will confine affective situations to the circumstances in which 'mathematical content' is involved.

The teacher may pose situations in which, specifically, the students' beliefs are brought into play towards a concrete mathematical object. For example, the MDF1 reflects a situation being proposed by the teacher, in the first session of the lesson, to detect the students' beliefs about chance and random sequences.

> MDF1: Situation-problem specifically implemented so that the students' beliefs towards a specific mathematical object become evident.
> [Teacher's diary] I introduce the first activity, about the distinction between random and deterministic phenomena. It consists of cutting 20 more or less equal pieces of paper, or 20 balls. Once students are finished, they are told to place them randomly on the table.

Thus, in a dialogical environment and with manipulative material, the teacher observes the random dispositions of the objects on the tables and can ask questions to encourage reflection on them, while assessing the starting beliefs.

Affective practices

Affective practices are any action or affective manifestation that accompanies any mathematical practice. They can be manifestations about emotions, attitudes, beliefs or values about the objects put into play. Each of these affect expressions can vary in intensity throughout a practice or even disappear, giving rise to new manifestations. The great part of the affective trajectory remains hidden from the eyes of the teacher, because not all the affective states are manifested. Besides, it is not possible for a single person to observe the whole group to interpret small gestures or signs of every student.

Nevertheless, an observation record, as a classroom diary (Porlán and Martín, 1991), helps to collect data on which to reflect later. And, in addition, there are instruments that can be incorporated into the teaching practice to gather information about the affective domain. This is the case, for example, of the humour map of the problems (Gómez-Chacón, 2000b), which the authors have used in previous research (Beltrán-Pellicer, 2015; Beltrán-Pellicer & Godino, 2017). Each student draws pictograms among 14 possible (or makes marks on a worksheet), to express what they feel during the process of solving a problem or task. The 14 pictograms represent 14 emotions: curiosity, great, boredom, indifference, mental block, despair, tranquillity, animation, haste, bewilderment, wracking my brain, pleasure, fun and trust. This map pursues a double purpose. On the one hand, it is a meta-affective practice, in which students become aware of their own emotional dynamics when trying to solve a mathematical situation. On the other hand, the information can be collected by the teacher, so that it can highlight affective facts that have allowed progress in the resolution and reflect on those that block or hamper progress. This tool was introduced in one of the experiences we refer to, as shown in MDF2:

> MDF2: Introduction of the humour map. Considering the affective domain.
> [Teacher's diary] […] let's do the activities with the humour map of the problems. They are surprised when I give them the worksheets and proceed to explain them calmly. Likewise, I explain that they should mark those states with which they feel most identified.

Intervening and emerging objects

Although the categorization of the affective domain in emotions, attitudes and beliefs is accepted by the research community, to which values can be added, the meaning of such constructs is still a matter of controversy. To describe and catalogue the affective objects that intervene or emerge in mathematical practices, we will use the tetrahedral model proposed by DeBellis and Goldin (2006), in which the meanings of the affective constructs are described as follows (p. 135):

- Emotions: quickly changing feelings experienced in a conscious way or occurring pre-consciously or unconsciously during mathematical (or other) activity. Emotions vary from mild to intense and are locally and contextually immersed.
- Attitudes: describe orientations or predispositions towards certain sets of emotional sensations (positive or negative), in particular (mathematical) contexts. This differs from the more common view of attitudes as predispositions toward certain patterns of behaviour. Attitudes are moderately stable, implying an interactive balance between affection and cognition.
- Beliefs: they imply the attribution of some kind of truth or external validity to the system of propositions or other cognitive configurations. Beliefs are often highly stable, largely cognitive and structured, in which emotions and attitudes intersect with them, contributing to their stabilization.
- Values: including ethical and moral components, refers to personal truths or commitments deeply appreciated by individuals. They help to motivate long-term decisions or set short-term priorities. They can be highly structured, building value systems.

Given the interaction with the cognitive domain, it may be convenient to consider, as a category of affective objects, the various modes of expression of the affects: gestures, terms of ordinary language, etc. (Álvarez, 2012), which would constitute the ostensible facet of affections. Emotions, attitudes, beliefs and values are relative to mathematical situations and practices, and to the distinct primary mathematical objects. It makes sense, therefore, to research the affective components towards the demonstrations, the procedures, the representations, etc. Figure 1 summarizes the primary affective-cognitive categories.

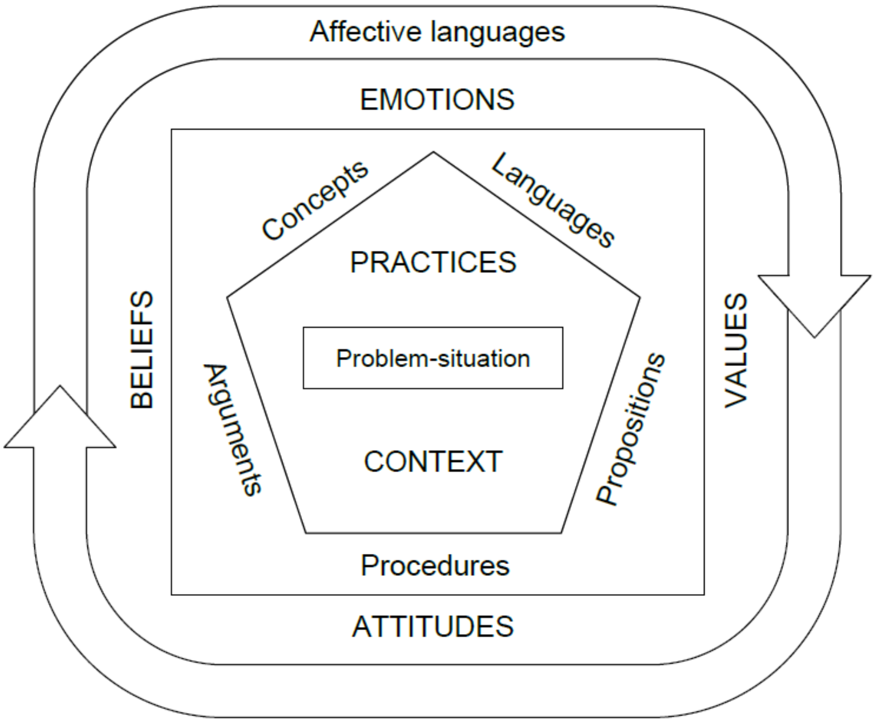


Figure 1. Affective and primary cognitive categories

The characteristics of affective languages, which could be considered as a fifth category of affective objects, expand the semiotic registers and representations that emerge from the practices, since much of the affective charge is expressed nonverbally, within a system of information transmission, in which each element is interpreted by the different agents involved (teacher, students).

Emotions, therefore, can arise as an instantaneous emotional response to a sensorial stimulus, which may have a mathematical character (a field of problems) or not (going to school). Although this distinction seems trivial, the origin of emotions is complex to interpret. Consider the MDF3, in which the annotation in the teacher's diary indicates that his students show nervousness and agitation because of the proximity of a written test:

> MDF3: Nervousness because of the proximity of a written evaluation test.
> [Teacher's diary] I tell the students that they already know the subject and that the examination date will be in 7 days. They complain, arguing that they have another test that day. Most of them, who were quite distracted and talking about other things, join in the discussion.

Whereas it is clear that this is an emotional response, instantaneous, to a particular stimulus (the teacher announcing the examination date), the actual origin of the emotion could be based on specific beliefs about math tests or more general beliefs about school. Other times, it is easier to identify the source of those emotions, although it is not possible to establish absolute certainty, which would require more data collection tools. For example, the MDF4 describes an emotional reaction that could be named curiosity, to comments of the teacher that try to confront the students' beliefs about random experiments with a specific mathematical fact, in this case, the stability of the relative frequencies. Some students hold this emotion, which leads to an attitude of interest:

> MDF4: Emotion that arouses interest.
> [Teacher's diary]. I briefly introduce the stability of the relative frequencies, summarizing the previous day activity (coin tosses). I ask them about the results of the coins, 'to what number the percentage comes up', to which they say that 'one goes up and the other one goes down'. I see that at least A7 looks interested. I suggest them to think about what would happen if the coins were tossed 1000 times.

The feedback system formed by the different components of the affective domain is put into play in any type of situation. The MDF5 shows how, when facing a problem-situation, some students show mental block emotion, while others start from a passive attitude, which they have been able to reach from the previous emotion or from causes unrelated to the situation. One of the teacher objectives in these cases is to intervene in the feedback loop (attitude-emotional) to promote progress in the teaching-learning process:

> MDF5: Emotion and attitude within a situation-problem.
> [Teacher's diary] I see that there are students who are finishing, but also some who go slowly, either because they do not want to do it (case of A6) or because they get stuck. In the case of A6, I urge her to finish it.

The students to whom the teaching-learning sequences are directed present beliefs about the mathematical objects that make up the trajectory of the instructional process. In the case of probability, its different meanings (Batanero, 2005) must be negotiated from personal belief systems, as seen in MDF6, where students, used to solve similar problem-situations with other procedures, are reluctant to use a new one:

> MDF6: Belief about the procedure to solve a situation-problem.
> [Teacher's diary] I see that some of them have already reached an exercise in which they get stuck, and I introduce the tree diagrams, as a helpful way to solve it. But I notice that some students are still trying to do it without using the diagram and without success.

At other times, in a dialogic interaction environment, it is the teacher who decides to inquire directly about the students' beliefs. The MDF7 shows an example of this, in which the teacher asks about the distinction between random and deterministic phenomena, an issue that relates to the perception of chance:

> MDF7: Beliefs about random phenomena.
> [Teacher's diary] They have no problem to specify the sample space of these experiments (balls extraction, pushpin that is thrown to the ground) or to distinguish if they are random or deterministic. I take the time to ask if predicting tomorrow weather will be random or deterministic.

Ethical and moral values differ from beliefs in which, while the latter constitute judgments of subjective truth from the logical or empirical point of view, values refer to purely personal choices (that which is good, or desirable) (Goldin, 2002). However, belief systems and value systems are closely related, and at times, it is difficult and inoperative, to isolate them. The MDF8 exemplifies how the commitment to the study process (a personal choice that constitutes a value) influences the learning trajectory, directly reducing the effective teaching time:

> MDF8: Value about commitment to the study process.

> [Teacher's diary] They take a long time to come from recess. I must raise my voice and show myself authoritative, so they can take out a notebook and a book. A5 takes even longer, speaks and laughs with his mates and has not brought the book.

On the other hand, affective languages deserve special attention, and this is reflected in the key place reserved for them in Figure 1. Language, in its different registers, constitutes not only a communicative vehicle, but, being formed by signs which are constantly interpreted, is a tool of signification. In the case of the affective domain, non-verbal communication plays a fundamental role (Johnson, 1999; Knapp, Hall, & Horgan, 2013). In the same way that pupils' productions, both written (also in their different registers) and verbal, provide indicators about the cognitive domain, the transmission of much of the affective information is done through facial expressions, gestures, postures, movements, etc.

Harris and Rosenthal's (2005) meta-study shows how students improve in certain facets when the teacher's non-verbal language includes immediacy signs, such as gesturing when speaking, not sitting behind his desk, looking at students while talking, smiling, using a tone that is not monotonous, etc. Thus, students show interest in the course and the teacher, pay attention and have the perception that they have learned a lot in class (Rocca, 2004). Likewise, the results of his study also show correlations between the teacher's non-verbal language and students' cognitive performance, although this is something that is under study (Witt, Wheeless, & Allen, 2004). All these affective languages match interaction patterns that can be encompassed into one of the following three dimensions (Rompelman, 2002): opportunity to respond in a climate of trust, possibility of feedback, and consideration towards people (respect). Harris and Rosenthal (2005) also point out the difficulty of investigating empirically in the classroom environment, due to the apparatus required to capture all non-verbal information.

Other authors agree in this regard. Mitchell (2013) points out that, given the positive relationship between the teacher's non-verbal language and student attitudes, it is important that the teacher is not only enthusiastic about content, but should also show that enthusiasm to have a positive impact in the learning of the students. This influence of the teacher in the students' beliefs towards mathematics and that, in the end, influence the other affective components, is evident in the essay excerpt shown here:

> Mathematics was never my strong point, rather, my Achilles heel. I came to hate them when I was in High School, despite this, little by little I can understand them thanks to my perseverance and dedication.
> Depending on the syllabus, I pay more attention when I find it interesting, although if I get bored I disconnect.
> Also depending on the teacher, which plays a fundamental role in making learning easier.

In the excerpt, the student mentions the importance of certain attitudes (constancy, perseverance) and of the emotions that are awakened by some content (boredom), as well as the role that the teacher plays to encourage interest.

Contextual dualities

Next, we analyse the four types of affective entities of the tetrahedral model of DeBellis and Goldin (2006) from the point of view of the five pairs of contextual dualities introduced in the OSA: personal - institutional, expression - content, ostensive - non-ostensive, intensive - extensive, unitary - systemic. We consider that this analysis allows to articulate aspects of the affective domain that are treated in a non-systematic or tangential way in the literature. Figure 2 synthetizes these dualities in a single diagram, to which we will refer later.

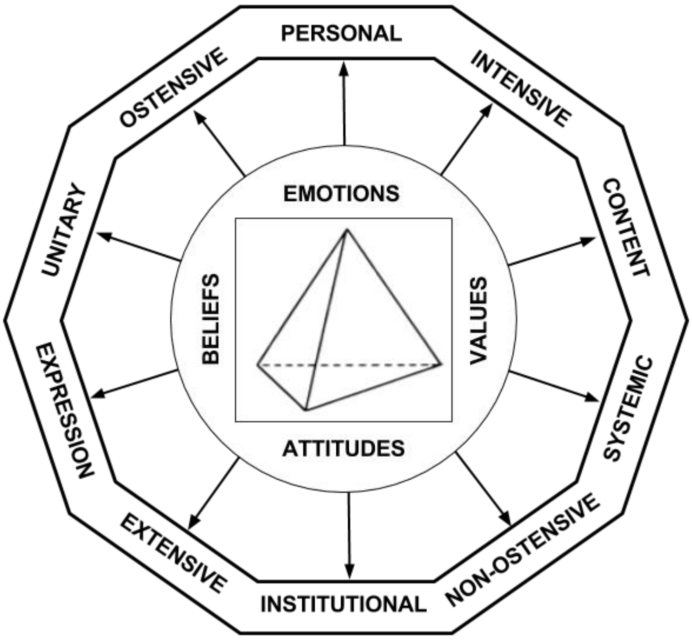


Figure 2. Contextual dualities for the affective facet

Personal - institutional

Affective objects and processes are usually considered as psychological entities, which refer to more or less stable states or mental traits, or dispositions for the action of individual subjects. However, from the educational point of view, the achievement of affective states that interact positively with the cognitive domain must be object of interest by the teacher, that is, by educational institutions. The fact that there is research on affectivity means that it is possible to identify phenomena, regularities and shared conceptualizations that confer a certain degree of objectivity to the affects and their influence on learning. The affective domain therefore carries an institutional facet and is concretized in rules of an affective nature that condition the teacher work.

The personal - institutional distinction, both for the cognitive and the affective facets, allows us to focus attention on the dialectic between these dimensions, therefore, to become aware of the various institutional conditions in which affective phenomena take place.

Different curricular regulations, such as the Spanish one (MECD, 2014) establish orientations from the affective point of view, mainly as far as attitudes and values are concerned. Thus, the aim is to promote certain attitudes (understood almost as competences or skills) in the problem-solving contexts. Examples of this are the following evaluation standards for 3rd year of Secondary Education (14-15 years old):

> 8.1. He/she develops adequate attitudes for work in mathematics: effort, perseverance, flexibility and acceptance of reasoned criticism.
> 8.2. He/she faces the resolution of challenges and problems with the precision, care and interest appropriate to his/her educational level and the difficulty of the situation.
> 8.3. He/she distinguishes between problems and exercises and adopts the appropriate attitude for each case.

> 8.4. He/she develops curiosity and inquiry attitudes, along with habits of asking questions and seeking adequate answers, both in the study of concepts and in problem- solving. (MECD, 2014, p.391)

The institutional dimension is essentially static. However, these norms are interpreted in the first instance by the teacher, since while he/she is planning each lesson, the corresponding curricular orientations must be incorporated, being confronted with its own systems of beliefs and values. In a second level, when the teacher effectively implements the sessions, the emotions (instant affective states) towards that group of students and with those specific contents interact with the attitudes, beliefs and values. This forms a system that is framed in the personal dimension (of the teacher) and that is fed again and again. That is, emotions arise as a representation (ostensible or not) of their attitudes, beliefs and values. And these latter categories of entities are reinforced or modified by the persistence of those emotions over time. And the same goes for students when they interact with institutional rules.

On the other hand, it is in the personal dimension where other interactions take place, in the form of semiotic functions, between affective entities and other types of entities, such as those of the cognitive plane (Whitson, 1997). These interactions are nothing more than interpretation processes, and therefore meaning processes, elements from a domain (e.g., epistemic, cognitive) that play the role of signs for the other domain (e.g., affective).

Figure 3 shows the resolution of a problem of divisibility, by a student, through two different procedures, which can be interpreted as an expression of the personal-institutional duality. That is, the first of the procedures is the one that this student would use naturally, and then adds a second procedure because he/she thinks that it is the required one for this task.

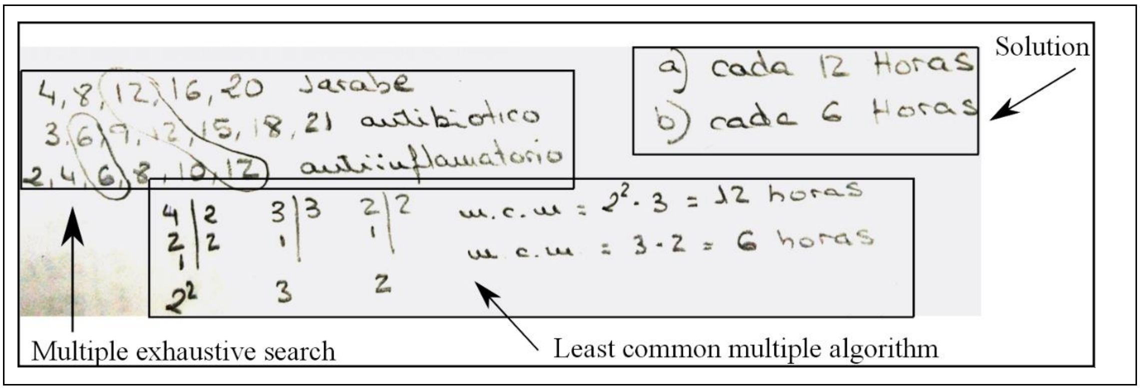


Figure 3. Resolution of a problem by two different procedures.

This duality can also be easily identified in devolution moments (Brousseau, 1997), when the teacher shares his/her responsibility with the students, hoping that they progress autonomously. The MDF9 reflects this duality:

> MDF9: The teacher expects one thing and the students do another.
> [Teacher's diary] There are students who have written nothing at all in the notebook, neither the problems we have corrected at the beginning of the class, nor those of the worksheet (A6, A16 and A18). I urge them to work.

Expression - content (affective semiotic functions)

The affective objects cannot be conceived as isolated entities but give rise to interpretation processes by the subjects or the institutions. In other words, they intervene as antecedents and consequents of semiotic functions. Goldin (2000, p. 211) attributes a representational valence to affect: 'Note that the very notion of code here suggests that something is being encoded, that affective configurations can signify or represent information.' This component of Goldin's theory of representations finds in the OSA a natural fit through the notion of semiotic function, as we will see below.

The (pragmatic) meaning of an affect can be defined as the system of affective practices in which that affect participates in a relevant way. That is, the effects or consequences that an affect has in the accomplishment of a mathematical practice. Another use of the term affective meaning can be of referential type when the expression or antecedent of the semiotic function is an affective linguistic expression and the consequent or meaning is the affect to which it refers. One can speak, therefore, of emotional meaning, attitudinal meaning, etc. of an affective expression.

In this way, authors such as Flavier, Bertone, Hauw and Durand (2002) identify the object of Peirce's sign as the student's concern about a given situation, which opens the possibility of subjective judgments that depend on previous experience. The representamen or sign, on the other hand, is the element of the situation under consideration which, in our case, would be each of the categories of mathematical objects. To complete the sign triadic conception, the interpretant corresponds to the mobilization of knowledge during the situation.

The notion of semiotic function is also useful as an entity that connects or relates the affective entities themselves, both from a referential and an operational point of view. It also connects the affective entities to the cognitive and epistemic entities. The difference with the personal - institutional duality is that the epistemic content is interpreted here, not the intentionality of the practice, as exemplified by the MDF10:

> MDF10: The statement of a problem situation provokes an alternative analysis.
> Three safety devices A, B and C have been installed. If device A fails, device B is started automatically, and, if this fails, C is started. The failure probability of A is known to be 0.1. The probability that B will work is 0.98 and the probability that C fails is 0.05. Calculate the probability that everything works fine.
> [Teacher's diary] A16 makes an interesting and funny remark at the same time. If device B is the one most likely to work, why not put it first.

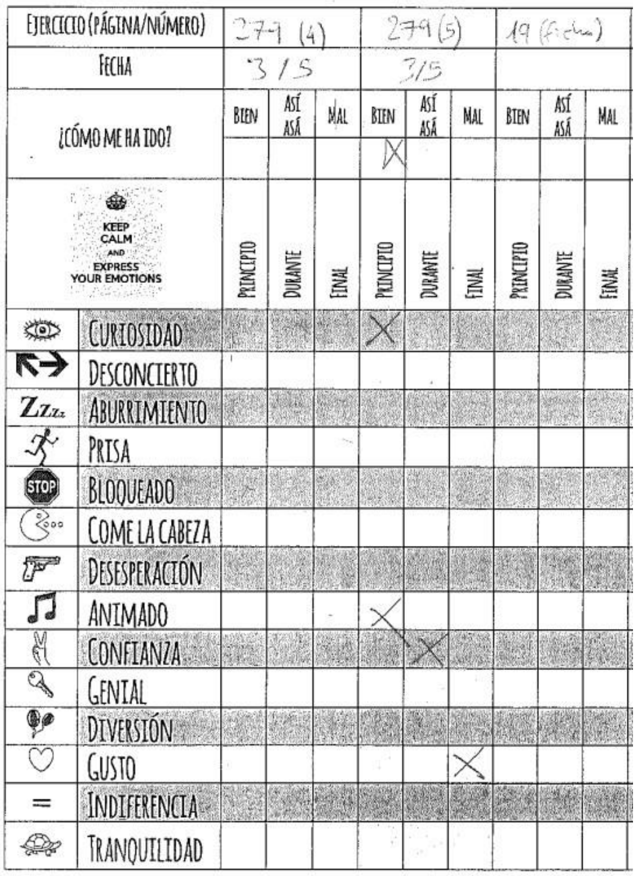
The exchange of information between representational systems mentioned by Goldin (2000, p. 211) and DeBellis and Goldin (2006, p. 133) can also be interpreted with the notion of semiotic function. In this way, the meanings encapsulated in each of the representations of the affective domain are related, through semiotic functions, to representations of other domains such as verbal, imagistic, formal and the planning and executive system (Goldin & Kaput, 1996). And vice versa, a function that has as starting domain an imagistic representation, for example, can transfer that meaning to the arrival domain, evoking an affect related to the meaning that is encapsulated by the function.

Ostensive – non-ostensive

Affects are mental entities (or ideals), that is, not ostensible by nature (they are not directly perceptible). But they manifest themselves through gestures and specific expressions, that is, through ostensible manifestations. The work of DeBellis and Goldin (2006) studies how to infer the internal entities from the available observations, as well as the exchanges of information (interactions) between the affective representation system and the other systems of representation that intervene in problem-solving situations.

Since mathematical objects (concepts, procedures, arguments and propositions) require for their manifestation of linguistic elements, language itself (in its various manifestations and registers) is considered within the OSA as mathematical object, or rather, as an object that intervenes in mathematical practice (Font, et al., 2013). The identification of affective objects presents even greater difficulty, if at all. In an analogous way to mathematical objects, their knowledge will only be possible from their external manifestations. Now, meanings linked to the individual's affective states often remain unconscious or preconscious, difficult to verbalize by the individual who experiences them, and subject to a complicated interpretation by outside observers (DeBellis & Goldin, 2006, p. 133). Affective signs are small gestures in the body language, changes in the voice intonation, sighs, facial expressions, etc. whose precise meaning is, at least ambiguous. However, their effectiveness as a communication system is evident, because they provoke emotional reactions in other subjects, who interpret these signs, often in an unconscious or preconscious way.

The use of specific instruments, such as the humour map (Gómez-Chacón, 2000b) that was used in one of the experiences that we use as a reference, does not completely solve this problem (Figure 4). The description of the emotional states by the teacher when introducing this tool for the first time to the students, even when it helps to establish reference meanings, is not infallible. The students' self-knowledge of their affective processes, on the other hand, is variable, which justifies the feeling of the same emotion towards a task (e.g., 'I can't solve this problem', 'I don't know how to do this'), some students verbalize them as despair and others as *mental block*.



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¿Cómo me ha ido?	Bien	Así asá	Mal	Bien	Así asá	Mal	Bien	Así asá	Mal
				X					
KEEP CALM AND EXPRESS YOUR EMOTIONS	Principio	Durante	Final	Principio	Durante	Final	Principio	Durante	Final
Curiosidad				X					
Desconcierto									
Aburrimiento									
Prisa									
Bloqueado									
Come la cabeza									
Desesperación									
Animado				X					
Confianza					X				
Genial									
Diversión									
Gusto						X			
Indiferencia									
Tranquilidad									

Task (page/number)	279 (5)		
Date	3/5		
How did it go?	Right	Fair	Wrong
	x		
	Start	During	End
Curiosity	x		
Bewilderment			
Boredom			
Haste			
Mental block			
Wracking my brain			
Despair			
Animation	x		
Trust		x	
Great			
Fun			
Pleasure			x
Indifference			
Tranquillity			

Figure 4. Sample of emotional data collection (Beltrán-Pellicer and Godino, 2017), based on the humour map of the problems (Gómez-Chacón, 2000b)

Intensive – extensive

Goldin (1988) introduced the distinction between local and global affect. Local affect refers to the changing and instantaneous affective states that appear during problem-solving situations, constituting a system of internal representation, on the same level as imagistic representation, formal notations, verbal representations, and the meta-system formed by planning and executive control (DeBellis & Goldin, 1991, p.29). Global affect, on the contrary, is constituted by attitudes, which rely directly on systems of beliefs and values. Like local affect, global affect can be expressed in any type of situation, but its entities are not so changeable or easily modified.

Local affect is formed by the emotions that are experienced within the different problem-situations that are proposed to the students. It includes, therefore, manifestations (when the emotions are externalized) or feelings (when the instant affective entities remain internalized) of ephemeral and particular character, in a particular moment. If individuals experience the same affective states when facing similar situations, these are reinforced, configuring a system (global affection) in which already come into play attitudes, beliefs and even values. In other words, it is a process of generalization, so that when the teacher proposes a situation that evokes tasks and activities already lived, the student performs a particularization action, because the emotions will depend on their own attitudes and beliefs, which in turn are formed as a generalization of emotions.

Unitary – systemic (reification, decomposition)

A characteristic affective trait of a person (e.g., a negative attitude towards mathematics) can be interpreted as the result (reification) of a sequence of negative affective experiences in relation to mathematical learning. The research about its origin and the design of change strategies may require analysing and decomposing this trait into partial aspects.

The contextual dualities apply to each of the affective (and cognitive) entities of the model represented in Figure 3. The unitary - systemic duality arises when considering the different objects toward which emotions, attitudes, beliefs and values are directed. The interrelation between them all forms the affective system of a person.

In the field of mathematics education, McLeod (1992) distinguishes different types of beliefs: about mathematics, about self, about mathematics teaching and about the social context. The same happens with attitudes, being possible to distinguish between attitudes towards mathematics (interest, satisfaction, curiosity, etc.) as well as mathematical attitudes (flexibility in the choice of techniques and strategies, critical spirit, etc.) (Callejo, 1994; Gil, Blanco, & Guerrero, 2005).

The teacher should pay attention to the unitary manifestations of belief systems and the attitudes that emerge from them, taking note of those emotions and instantaneous affective responses that favour appropriate mathematical attitudes for the different types of situations that take place in the teaching-learning processes.

Dynamics of affect

So far, we have presented a static view of affect and its relationships with the cognitive - epistemic domain. The dynamics of affect must be researched within the instructional processes. Some theoretical notions have been introduced by the OSA that can help the study of the dynamic aspects of affect in mathematics education. It is the case of the notions of didactical configuration and didactical trajectory. A didactical configuration is any segment of didactic activity (teaching and learning) carried out between the beginning and the end of a task (situation – problem). So, this includes the students' and the teachers' actions, as well as the resources planned or used to carry out the task. The sequence of didactic configurations makes up a didactic trajectory. In any didactic configuration there is: a) an epistemic configuration (system of practices, objects and institutional mathematical processes required in the task), b) an instructional configuration (system of teacher/learner functions and instructional means which are used in addition to the interaction between the different components) and c) a cognitive-affective configuration (system of practices, objects and personal mathematical processes that describe the learning and the affective components which accompany it).

From an instructional point of view, the affect in mathematics education must be analysed, planned, implemented and evaluated, like the rest of the facets. Research on the relation between the affective domain and mathematics focuses usually on interactions with the cognitive domain. It seems necessary, however, to consider also the epistemic component, the interaction patterns in the classroom, the use of resources, as well as the other ecological conditions that determine the study processes in the educational institutions (curriculum, social and political factors, etc.).

The identification of the affective trajectory and how it interacts with the epistemic configurations and with the cognitive domain, allows the teachers to use this information to suggest strategies for problem-solving to the students (Caballero, Cárdenas and Gómez, 2017). An emotional state that may seem a priori negative, such as mental block or despair, may be the beginning of an affective trajectory that catalyses a cognitive sub-trajectory that leads to the use of the necessary heuristics to solve the corresponding problem-situation. This sub-trajectory would interact again with the affective domain, in a kind of continuous feedback, favouring the appearance of positive emotions. These dynamics can be interpreted in a representational sense:

Roughly speaking, it means that the states of emotional feeling carry meanings for the individual. They encode and exchange information in interaction with other internal systems of representation, in a way essential to mathematical understanding and problem-solving performance. (DeBellis & Goldin, 2006, p. 133)

That is, from the point of view of Peirce's semiosis (Peirce, 1931-1935), the representations (signs) of the entities of a domain (objects) can be interpreted with different meanings if they are decoded from another domain. Di Martino and Zan (2011) propose the Three-dimensional Model for Attitude towards mathematics (TMA) as a way of interpreting the relationship between emotions and beliefs. This relationship is complex, and part of the identification of three attitudinal dimensions (p. 476):

- Emotional disposition towards mathematics, concisely expressed with 'I like/dislike mathematics'.
- Perception of being/not being able to succeed in mathematics, what often is called 'perceived competence' (Pajares & Miller, 1994), concisely expressed with: 'I can do it/I can't do it'.
- Vision of mathematics concisely expressed with 'mathematics is…'.

The study carried out by Di Martino and Zan (2011), based on autobiographical essays, also establishes a series of relations between these three dimensions, which enriches previous research (Callejo, 1994; Gil, Blanco, & Guerrero, 2005; McLeod, 1992).

- A negative emotional disposition (emotion, attitude) is related to a biased view of mathematics (belief).
- A negative emotional disposition (emotion, attitude) is also related to the level of self-perceived competence (belief).
- The view of mathematics (belief) is strongly related to the level of perceived competence (belief).

This system of relationships does not have to be unidirectional or dependent. The authors point out, on the other hand, the possible utility of the TMA model to design situations that favour appropriate attitudes and beliefs. The dynamics of affect are directly related to the interactional facet of the instructional processes, giving rise to interaction patterns that may promote or interfere learning (García, 2009).

Final reflections

The affective facet is perhaps the most complex to evaluate within the six facets that make up the didactical suitability in the OSA, and an initial proposal was suggested in Godino (2013). Didactical suitability is understood as the degree to which an instructional process (or a part of it) combines certain characteristics in order to be classified as suitable (optimal or appropriate) for the adaptation between the personal meanings obtained by students (learning), and the intended or implemented institutional meanings (teaching), taking into consideration the circumstances and the available resources (environment). The didactical suitability notion is broken down into six specific facets (epistemic, cognitive, affective, interactional, mediational and ecological suitability) (Breda, Pino-Fan, & Font, 2017). This notion is being widely used in research on teachers' education as a tool that supports teacher's reflection on their own practice (Breda, Font, & Pino-Fan, 2018).

The literature review clearly indicates that the relationship with the students' perception of their own learning influences their own progress. There is also a certain correlation between affection and degree of cognitive performance, although more studies are needed in this line (Gómez-Chacón, 2017).

The degree of affective suitability, understood as the degree of involvement, interest and motivation of the students, in a process of teaching and learning of mathematics can be assessed considering the following criteria (Table 1).

This work constitutes an advance within the OSA theoretical framework, when applying their primary categories and their ontology to the affective domain of the teaching-learning processes. The pertinence of the study is reflected both in the articulating nature between representational systems of theoretical models and in the revision of the original criteria on affective suitability proposed by Godino (2013).

Table 1: Affective suitability components and criteria of a study process in mathematics.

COMPONENTS	CRITERIA
Languages	1. Attention is paid to non-verbal language to foster immediacy.
Emotions	1. Qualities of aesthetics and precision of mathematics are highlighted. 2. Specific moments along the sessions are scheduled so that students can express their emotions towards the proposed situations.
Attitudes	1. Self-esteem is promoted, avoiding rejection, phobia, fear of mathematics. 2. Participation in activities, perseverance, responsibility, etc. is promoted to foster a mathematical attitude. 3. Argumentation is favoured in situations of equality. The value of an argument does not depend on who says it.
Beliefs	1. The beliefs about mathematics, about the meta-cognition of students, about the teaching of mathematics and about the social context in which they develop learning are explored and considered.
Values	1. The value and usefulness of mathematics attributed by students in daily and professional life are explored and considered.
Interrelation with other domains / facets	1. The affective component is planned in the teaching-learning process. 2. Positive emotions are related to mathematical attitudes and to the successful resolution of tasks, fostering the emotional reflection of students in this regard.

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